1314-4-4-Notes-exp-log-equations

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4.4: Exponential and Logarithmic Equations

An *exponential equation* is an equation containing a^x in some form.
A *logarithmic equation* is an equation containing a logarithm.

This section doesn't really contain any new facts, but there are many types of problems, so we'll just do a whole bunch of examples.

If it is possible to write your answer without logarithms, you should do so. If this is not possible, write your answer in terms of natural logarithms.

Example 1: Solve $3^x = 9$.

Example 2: Solve $2^x = 7$.

Example 3: Solve $6 \cdot 5^{-6x} = 3$.

$$\frac{\cancel{6} \cdot 5^{-6x}}{\cancel{6}} = \frac{3}{6}$$

$$5^{-6x} = \frac{1}{2}$$

$$\ln 5^{-6x} = \ln\left(\frac{1}{2}\right)$$

$$-6x \ln 5 = \ln\left(\frac{1}{2}\right)$$

$$\frac{\cancel{-6}x \cancel{\ln 5}}{\cancel{-6} \cancel{\ln 5}} = \frac{\ln\left(\frac{1}{2}\right)}{-6\ln 5}$$

$$x = -\frac{\ln\left(\frac{1}{2}\right)}{6\ln 5}$$

$$= -\frac{\ln 1 - \ln 2}{6\ln 5}$$

$$= -\frac{0 - \ln 2}{6\ln 5} = -\frac{-\ln 2}{6\ln 5}$$

$$= \frac{\ln 2}{6\ln 5}$$

$$\boxed{\text{sol'n Set: } \left\{\frac{\ln 2}{6\ln 5}\right\}}$$

Recall

$\log_a P^n = n \log_a P$

$\log_a\left(\frac{P}{Q}\right) = \log_a P - \log_a Q$

$\ln 1 = 0$ because $e^0 = 1$

Example 4: Solve $2^{7-3x} + 1 = 11$.

Example 5: Solve $2e^{12x} = 17$.

Isolate the exponential: $\frac{\cancel{2}e^{12x}}{\cancel{2}} = \frac{17}{2}$

$$e^{12x} = \frac{17}{2}$$
$$\ln(e^{12x}) = \ln\left(\frac{17}{2}\right)$$
$$12x = \ln\left(\frac{17}{2}\right)$$
$$\frac{12x}{12} = \frac{\ln\left(\frac{17}{2}\right)}{12}$$
$$x = \frac{\ln\left(\frac{17}{2}\right)}{12} = \frac{\ln 17 - \ln 2}{12}$$

Recall:
$$\log_b b^x = x$$
$$b^{\log_b x} = x, \text{ for } x > 0$$
$$\ln e^x = \log_e e^x = x$$

Sol'n Set: $\boxed{\left\{\frac{\ln 17 - \ln 2}{12}\right\}}$

Example 6: Solve $e^{9-2x} - 1 = 17$.

$$e^{9-2x} - 1 = 17$$
$$+1 \quad +1$$
$$e^{9-2x} = 18$$
$$\ln e^{9-2x} = \ln 18$$
$$9 - 2x = \ln 18$$
$$-9 \quad -9$$
$$-2x = -9 + \ln 18$$
$$x = \frac{-9 + \ln 18}{-2}$$

Sol'n Set: $\boxed{\left\{\frac{-9 + \ln 18}{-2}\right\}}$

OR $\boxed{\left\{\frac{9 - \ln 18}{2}\right\}}$

Note: Same as $\frac{-9 + \ln 18}{-2} \cdot \frac{-1}{-1}$

Example 7: Solve $\frac{4}{6 - e^{2x}} = -5$.

$$\frac{\cancel{6 - e^{2x}}}{1} \cdot \frac{4}{\cancel{6 - e^{2x}}} = -5(6 - e^{2x})$$
$$4 = -5(6 - e^{2x})$$
$$4 = -30 + 5e^{2x}$$
$$34 = 5e^{2x}$$
$$\frac{34}{5} = e^{2x}$$
$$\ln\left(\frac{34}{5}\right) = \ln e^{2x}$$
$$\ln\left(\frac{34}{5}\right) = 2x$$
$$\frac{\ln\left(\frac{34}{5}\right)}{2} = \frac{2x}{2} \Longrightarrow x = \frac{\ln\left(\frac{34}{5}\right)}{2} = \frac{\ln(34) - \ln(5)}{2}$$

used $\ln\left(\frac{P}{Q}\right) = \ln P - \ln Q$

Sol'n Set: $\boxed{\left\{\frac{\ln 34 - \ln 5}{2}\right\}}$

Recall: Log properties:

$$\log_a(PQ) = \log_a P + \log_a Q$$

$$\log_a\left(\frac{P}{Q}\right) = \log_a P - \log_a Q$$

Example 8: Solve $e^{2x} + 4e^x = 12$.

$$(e^x)^2 + 4(e^x) - 12 = 0$$

$u = e^x \Rightarrow$ $u^2 + 4u - 12 = 0$

$$(u+6)(u-2) = 0$$

$u + 6 = 0$	$u - 2 = 0$
$u = -6$	$u = 2$
$e^x = -6$	$e^x = 2$
$\ln e^x = \ln(-6)$	$\ln e^x = \ln 2$
$x = \ln(-6)$	$x = \ln(2)$

can't have ln of negative, so throw out $x = \ln(-6)$.

Sol'n Set: $\boxed{\{\ln 2\}}$

Example 9: Solve $3^{2x+4} = 5$.

Example 10: Solve $4^x = 5^{2x-7}$.

Recall: $\log_a P^n = n \log_a P$

$$4^x = 5^{2x-7}$$

$$\ln 4^x = \ln 5^{2x-7}$$

$$x \ln 4 = (2x - 7)\ln 5$$

$$x \ln 4 = 2x\ln 5 - 7\ln 5$$

$\quad -2x\ln 5 \qquad -2x\ln 5$

Get terms with x on one side; terms without x on other side!

$$x\ln 4 - 2x\ln 5 = -7\ln 5$$

$$x(\ln 4 - 2\ln 5) = -7\ln 5$$

$$\frac{x\cancel{(\ln 4 - 2\ln 5)}}{\cancel{\ln 4 - 2\ln 5}} = \frac{-7\ln 5}{\ln 4 - 2\ln 5}$$

$$x = \frac{-7\ln 5}{\ln 4 - 2\ln 5}$$

or $x = \frac{-7\ln 5}{\ln 4 - 2\ln 5} \cdot \frac{-1}{-1} = \frac{7\ln 5}{-\ln 4 + 2\ln 5}$

Sol'n Set: $\boxed{\left\{\frac{-7\ln 5}{\ln 4 - 2\ln 5}\right\}}$

or

$\boxed{\left\{\frac{7\ln 5}{-\ln 4 + 2\ln 5}\right\}}$

Note: When solving equations with logs, there are often many correct ways to write the answer.

So if you feel confident doing a problem, but your answer doesn't match the answer in the key, don't panic. Use the properties of logs to see if your answer can be rearranged to match the author's.

Example 11: Solve $5^{3x-7} = 8^{x+5}$.

$$5^{3x-7} = 8^{x+5}$$
$$\ln 5^{3x-7} = \ln 8^{x+5}$$
$$(3x-7)\ln 5 = (x+5)\ln 8$$
$$3x\ln 5 - 7\ln 5 = x\ln 8 + 5\ln 8$$
$$3x\ln 5 - x\ln 8 = 7\ln 5 + 5\ln 8$$
$$x(3\ln 5 - \ln 8) = 7\ln 5 + 5\ln 8$$
$$x = \frac{7\ln 5 + 5\ln 8}{3\ln 5 - \ln 8}$$

Sol'n Set:
$$\boxed{\left\{ \frac{7\ln 5 + 5\ln 8}{3\ln 5 - \ln 8} \right\}}$$

Example 12: Solve $e^x + 3 = 0$.

$$e^x = -3$$
$$\ln e^x = \ln(-3)$$
$$x = \ln(-3)$$

Impossible! logarithms can only be applied to positive numbers. You cannot apply a log to 0 or to a negative

$\boxed{\text{No Solution}}$

$\boxed{\text{Solution Set: } \varnothing}$ (the empty set)

Example 13: Solve $\log_2 x = 3$.

Example 14: Solve $\log x = -5$.

Example 15: Solve $\log_2(3x-1)-1=4$.

Isolate the log:

$\log_2(3x-1) = 5$

$2^5 = 3x-1$

$32 = 3x-1$

$33 = 3x$

$\frac{33}{3} = \frac{3x}{3}$

$11 = x$

Sol'n Set: $\boxed{\{11\}}$

or

$\log_2(3x-1) = 5$

$2^{\log_2(3x-1)} = 2^5$

$3x-1 = 2^5$

Example 16: Solve $\log_3(x-5)=2$.

$\log_3(x-5) = 2$

$3^2 = x-5$

$9 = x-5$

$9+5 = x \implies x = 14$

Sol'n Set: $\boxed{\{14\}}$

Example 17: Solve $\log_3 x + \log_3(x+2) = 1$.

$\log_3[x(x+2)] = 1$

$3^1 = x(x+2)$

$3 = x^2+2x$

$0 = x^2+2x-3$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x+3=0 \quad | \quad x-1=0$

$x=-3 \quad | \quad x=1$

Recall: Properties of Logs

$\log_a(PQ) = \log_a P + \log_a Q$

$\log_a\left(\frac{P}{Q}\right) = \log_a P - \log_a Q$

both only true when $P>0$ and $Q>0$

Check answers:

$x=1$: $\log_3 x + \log_3(x+2) = 1$

$\log_3 1 + \log_3(1+2) = 1$

$0 + \log_3 3 = 1$

$0 + 1 = 1$

$1 = 1$ ✓ It works!

$x=-3$: $\log_3(-3) + \log_3(-3+2) = 1$

Can't take log of a negative! Throw out -3

Sol'n Set: $\boxed{\{1\}}$

IMPORTANT: When solving logarithmic equations, you **must** "check" your solutions to make sure that don't cause the logs in the original equation to be undefined. These extraneous solutions generally happen when you combine separate logs into one log of a product.

Remember, you can't apply a logarithm to a negative number or zero!!!

Example 18: Solve $\ln(x+2)+\ln(x-4)=\ln 7$.

Use log property

$\log_a P + \log_a Q = \log_a(PQ)$

$\ln[(x+2)(x-4)] = \ln 7$

Raise e to both sides:

$e^{\ln[(x+2)(x-4)]} = e^{\ln 7}$

$(x+2)(x-4) = 7$

$x^2-2x-8 = 7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x-5=0$ | $x+3=0$

$x=5$ | $x=-3$

Check:

$x=5$: $\ln(5+2)+\ln(5-4)=\ln 7$

$\ln 7 + \ln 1 = \ln 7$

$\ln 7 + 0 = \ln 7$ ✓

$x=-3$: $\ln(-3+2)+\ln(-3-4)=\ln 7$

$\ln(-1)+\ln(-7)=\ln 7$

$\Rightarrow$ Throw out -3.

Sol'n Set: $\boxed{\{5\}}$

Example 19: Solve $\log_2(x^2-1)=3$.

Sol'n Set: $\{\pm 3\}$

Example 20: Solve $\log_2(x^2-x-2)=2$.

Sol'n Set: $\{-2, 3\}$

Example 21: Solve $\log_3(x+1)-\log_3(x-1)=2$.

Sol'n Set: $\left\{\frac{5}{4}\right\}$

Example 22: Solve $\log_2(x+5) = 4 + \log_2(x+1)$.

Example 23: Solve $5^{2x} - 7(5^x) + 12 = 0$.

Example 24: Solve $(\ln x)^2 + 7\ln x + 12 = 0$.

Example 25: Solve $e^{\ln x} = x$.

Example 26: Solve $\ln e^x = x$.