

1.5: Solving Quadratic Equations

1.5.1a

Solving Quadratic equations by factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic equation.

quadratic term $\rightarrow ax^2$
linear term $\rightarrow bx$
constant term $\rightarrow c$

Zero Product Property (Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB = 0$, then $A = 0$ or $B = 0$.

Example: Solve.

$$x^2 - 24 = 10x$$

$$x^2 - 10x - 24 = 0 \quad [\text{write in standard form}]$$

$$(x+2)(x-12) = 0 \quad [\text{factor the nonzero right side}]$$

$$\begin{array}{l} x+2=0 \quad \text{or} \quad x-12=0 \\ \underline{-2} \quad \underline{-2} \quad \quad \quad \underline{+12} \quad \underline{+12} \\ x = -2 \quad \quad \quad x = 12 \end{array}$$

[Apply zero product property. Set each factor equal to 0]

Solution Set: $\{-2, 12\}$

Check our solutions:

$$\begin{array}{l} x^2 - 24 = 10x \\ x = -2 \Rightarrow (-2)^2 - 24 = 10(-2) \\ 4 - 24 = -20 \\ -20 = -20 \quad \text{true!} \end{array}$$

$$\begin{array}{l} x^2 - 24 = 10x \\ x = 12 \Rightarrow (12)^2 - 24 = 10(12) \\ 144 - 24 = 120 \\ 120 = 120 \quad \text{true!} \end{array}$$

Note:

Expressions get simplified
(or factored or rearranged)
Equations get solved

Ex:

Solve.

$$x^3 - 6x^2 = -9x$$

+ 9x + 9x

$$x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x-3)(x-3) = 0$$

$$x=0 \quad \left| \begin{array}{l} \text{or} \\ x-3=0 \\ \quad \quad \quad \text{+3} \quad \text{+3} \end{array} \right| \quad \begin{array}{l} \text{or} \\ x-3=0 \\ \quad \quad \quad x=3 \end{array}$$

This is a cubic equation!
(not a quadratic)

Sol'n Set: $\boxed{\{0, 3\}}$

Ex:

$$4x + 70 = 2x^2$$

$$-2x^2 + 4x + 70 = 0$$

$$-2(x^2 - 2x - 35) = 0$$

$$-2(x-7)(x+5) = 0$$

$$-2 = 0 \quad \left| \begin{array}{l} \text{or} \\ x-7=0 \\ \quad \quad \quad x=7 \end{array} \right| \quad \begin{array}{l} \text{or} \\ x+5=0 \\ \quad \quad \quad x=-5 \end{array}$$

this factor
never
true
doesn't contribute
a solution

Sol'n Set:

$$\boxed{\{7, -5\}} \text{ or } \boxed{\{-5, 7\}}$$

$$\text{Ex: } 2x(x-4)(x+8) = 0$$

$$2x=0 \quad \left| \begin{array}{l} x-4=0 \\ \quad \quad \quad x=4 \end{array} \right| \quad \begin{array}{l} x+8=0 \\ \quad \quad \quad x=-8 \end{array}$$

$$\frac{2x}{2} = \frac{0}{2} \quad \left| \begin{array}{l} x=4 \\ \quad \quad \quad x=-8 \end{array} \right|$$

$$x=0 \quad \boxed{\{0, 4, -8\}}$$