P.2: Exponents and Scientific Notation

Recall: Properties (Laws, Rules) of Exponents

$$x^a x^b = x^{a+b}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(xy)^a = x^a y^a$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

Powers of Positive and Negative Numbers

Important:

$$-3^2 = -(3)(3) = \boxed{-9}$$

$$(-3)^2 = (-3)(-3) = \boxed{9}$$

Example:

$$(-2)^6 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+} = 4\cdot4\cdot4 = 16\cdot4 = \boxed{64}$$

$$(-2)^5 = \underbrace{(-2)(-2)}_{+}\underbrace{(-2)(-2)}_{+}\underbrace{(-2)}_{-} = 4\cdot4(-2) = \boxed{-32}$$

$$-2^6 = -(2)(2)(2)(2)(2)(2) = \boxed{-64}$$

$$-2^5 = -(2)(2)(2)(2)(2) = \boxed{-32}$$

=

Important: * If you raise a negative number to an even exponent, you get a positive result.

* If you raise a negative number to an odd exponent, you get a negative result.

$3^2 = 9$

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Ex: $(-3)^4 = \boxed{81}$

$(-3)^5 = \boxed{-243}$

On calculator:

$2^{13} = 8192$

Look for ^ or x^y or y^x

or $x^{\square}$

Know these powers:

$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

$9^2 = 81$
$9^3 = 729$

$10^2 = 100$
$10^3 = 1000$
$\vdots$
$10^7 = 10\,000\,000$

$6^2 = 36$
$6^3 = 216$

$7^2 = 49$
$7^3 = 343$

$8^2 = 64$
$8^3 = 512$

$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$

$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$

Ex: $-(-2)^4$

$= \boxed{-16}$

Ex: $-(-1)^{11}$

$= -(-1)$

$= \boxed{1}$

Ex: $\left(\frac{3}{4}\right)^2 = \boxed{\frac{9}{16}}$

Negative Exponents

Definition of a negative exponent:

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

Also: $\frac{1}{x^{-n}} = \frac{x^n}{1}$

Why? $\frac{1}{x^{-n}} = 1 \div x^{-n}$

$= 1 \div \frac{1}{x^n}$

$= 1 \cdot \frac{x^n}{1} = \frac{x^n}{1}$

Example: Simplify. $3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$

In these examples, write answers without negative exponents.

Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

Ex: $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}} = \frac{3^4}{2^4} = \boxed{\frac{81}{16}}$

OR $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \boxed{\frac{81}{16}}$ ← same!

Ex: $\frac{1}{4^{-2}} = \frac{4^2}{1} = \frac{16}{1} = \boxed{16}$

Ex: $2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$

Ex $(2x)^{-5} = \frac{(2x)^{-5}}{1} = \frac{2^{-5}x^{-5}}{1} = \frac{1}{2^5x^5} = \boxed{\frac{1}{32x^5}}$

Ex: $\frac{a^2}{a^{10}} = a^{2-10} = a^{-8} = \frac{a^{-8}}{1} = \frac{1}{a^8}$

Note: $\frac{x^a}{x^b} = x^{a-b}$

Better: $\frac{a^2}{a^{10}} = \frac{\cancel{a}\cancel{a}}{\cancel{a}\cancel{a}aaaaaaaa} = \boxed{\frac{1}{a^8}}$

Important Fact:

$x^0 = 1$, as long as $x \neq 0$

Here's why. $\dfrac{2^3}{2^3} = \dfrac{8}{8} = 1$

must be equal!

So $2^0 = 1$

From property of exponents,

$\dfrac{2^3}{2^3} = 2^{3-3} = 2^0$

Same approach would work for other bases, except 0 (can't divide by 0)

So 0^0 is undefined (indeterminate)

$2^5 = 32$
$2^4 = 32 \div 2 = 16$
$2^3 = 16 \div 2 = 8$
$2^2 = 8 \div 2 = 4$
$2^1 = 4 \div 2 = 2$
$2^0 = 2 \div 2 = 1$

Ex: Simplify, writing without negative exponents

$$\frac{x^{-3}y^4 z}{x^{-8}y^9 z^{-2}} = \frac{y^4 z \; x^8 \; z^2}{x^3 \; y^9} = \frac{x^8 y^4 z z^2}{x^3 y^9} = \frac{x^8 y^4 z^3}{x^3 y^9} = \boxed{\frac{x^5 z^3}{y^5}}$$

Ex: $$\left(\frac{2x^2}{y^{-4}}\right)^{-3} = \frac{2^{-3}x^{-6}}{y^{12}} = \frac{1}{2^3 x^6 y^{12}} = \boxed{\frac{1}{8x^6 y^{12}}}$$

P.2.5

Starting on Wed 9/11

Example: Simplify. Write answers without negative exponents.

$$\frac{(2x^2yz)(-x^4y^2z^3)^4}{(3xy)^2}$$

$$= \frac{2x^2yz(-1)^4x^{16}y^8z^{12}}{3^2x^2y^2}$$

$$= \frac{2x^2yz\cdot 1\cdot x^{16}y^8z^{12}}{9x^2y^2} = \frac{2x^{18}y^9z^{13}}{9x^2y^2}$$

$$= \boxed{\frac{2x^{16}y^7z^{13}}{9}} = \boxed{\frac{2}{9}x^{16}y^7z^{13}}$$

Note: $(4x^2y^3)^3$

$= (4)^3(x^2)^3(y^3)^3$

$= \boxed{64x^6y^9}$

Recall:

$x^ax^b = x^{a+b}$

$(x^a)^b = x^{ab}$

Ex: $$\frac{-3x^{-5}(x^{-1}y^3z^{-2})^{-4}}{(3x^2y^4)^{-2}(2xy^{-3}z)^4} = \frac{-3x^{-5}x^4y^{-12}z^8}{3^{-2}x^{-4}y^{-8}\cdot 2^4x^4y^{-12}z^4}$$

$$= \frac{-3}{x^5}\,\frac{x^4}{y^{12}}\,\frac{z^8\cdot 3^2x^4y^8}{\cdot 2^4x^4}\,\frac{y^{12}}{z^4}$$

$$= \frac{-3x^4z^8\cdot 9x^4y^8y^{12}}{x^5y^{12}\cdot 16x^4z^4} = \frac{-3\cdot 9x^8y^{20}z^8}{16x^9y^{12}z^4}$$

$$= \frac{-27y^8z^4}{16x} = \boxed{-\frac{27y^8z^4}{16x}}$$

P.2.6

Scientific Notation We write numbers so they have exactly one digit to the left of the decimal point, multiplied by a power of 10.

Scientific Notation is often used for very large and very small numbers.

Ex: Write 3254 in scientific notation.

$3254 = \boxed{3.254 \times 10^3}$

$3.\underbrace{254}_{3 \text{ slots}}$

Ex: Write 0.0001536 in scientific notation.

$0.0001536 = \boxed{1.536 \times 10^{-4}}$

$0.\underbrace{0001}_{4 \text{ slots}}536$

original decimal point

new decimal points

small numbers: negative powers of 10

big numbers: positive powers of 10

Ex. Write the number without scientific notation.

$2.5731 \times 10^{-2} = \boxed{0.025731}$

0.02.5731 move decimal 2 slot

new location

old location

Ex: write the number without scientific notation.

$7.62 \times 10^7 = \boxed{76\,200\,000}$ or $\boxed{76{,}200{,}000}$

$7.\underbrace{6200000}_{7 \text{ slots}}.000$

original

new location

Ex: On calculator:

$523 * 7^9$

Calculator has 2.1104936446 E10. This means $2.1104936446 \times 10^{10}$

2.1104936446^{10}

$1/23^{\wedge 8} \approx 1.2769600 54 \times 10^{-11}$

$\approx 1.277 \times 10^{-11}$

$\approx$ approximately equal