P4: Polynomials

An expression in which all variables are raised to non-negative integer exponents only.

Integers ..., -3, -2, -1, 0, 1, 2, 3, 4, ...

Examples of polynomials $5x^3 - 7x^2 + 12x - 3$

$\frac{3}{5}x^8 - \frac{1}{3}x + \frac{2}{7}$

Not polynomials: $5x^3 - 7x^{-2}$ (same as $5x^3 - \frac{7}{x^2}$)

$3 - 12x^{\frac{1}{2}} + x^{\frac{1}{3}}$ (same as $3 - 12\sqrt{x} + \sqrt[3]{x}$)

Standard form: Descending powers of x.

Ex: Write $3 - 12x - 7x^4 + 5x^2$ in standard form.

$\boxed{-7x^4 + 5x^2 - 12x + 3}$

Note $x^0 = 1$ (as long as $x \neq 0$)

Degree of a polynomial (for 1 variable): Largest exponent

Leading term: term with the largest exponent

Leading coefficient: Coefficient of the leading term.

Example: $3y^4 - 2y^6 - y^2 + 2y$

Leading term: $-2y^6$

Leading coefficient: -2

Degree: 6

Special terms for certain polynomials.

Polynomial with 3 terms: trinomial

Polynomial with 2 terms: binomial

Polynomial with 1 term: monomial

Ex: $4 - 2x^5$ is a 5^{th} degree binomial

Ex: $4x^3 - 2x^2 + 7$ is a 3^{rd} degree trinomial

Ex: $3x^4$ and $5x^2y^5z$ are both monomials.

For a polynomial with 2 or more variables, the degree is the largest total exponent on a term.

Example: What degree is $-2x^2y^4 + 3x^3y^3z - 2xy^8 + 7x^4 - 8xy$?

total exponent $2+4=6$

total exponent $3+3+1=7$

total exp is $1+8=9$

total exp 4

total exp is $1+1=2$

largest

$\boxed{\text{This is a } 9^{th} \text{ degree polynomial}}$

Multiplying polynomials:

Some Properties of Exponents:

$$x^a x^b = x^{a+b}$$
$$(x^a)^b = x^{ab}$$

Ex: $x^2 x^3 = (xx)(xxx) = x^5$

Ex: $(x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$

Adding and Subtracting Polynomials

Ex: $(3x^3 + 7x^2 - 6x) + (-5x^2 - 3x + 4)$

$= 3x^3 + 7x^2 - 6x - 5x^2 - 3x + 4$

$= \boxed{3x^3 + 2x^2 - 9x + 4}$

Ex: $(3x^2 - 7x + 1) - (-2x^2 - 8x + 5)$

$= 3x^2 - 7x + 1 - 1(-2x^2 - 8x + 5)$

$= 3x^2 - 7x + 1 + 2x^2 + 8x - 5$

$= 5x^2 + 1x - 4$

$= \boxed{5x^2 + x - 4}$

Properties (Laws, Rules) of Exponents

$$x^a x^b = x^{a+b}$$
$$(x^a)^b = x^{ab}$$
$$\frac{x^a}{x^b} = x^{a-b}$$

Ex: $\frac{x^5}{x^2} = \frac{\cancel{x}\cancel{x}xxx}{\cancel{x}\cancel{x}} = \frac{xxx}{1} = x^3$

Multiplying Polynomials

Multiplying a polynomial by a monomial:

(simplify/multiply)

Ex: $-2(4x^2 - 7x + 3)$

$= \boxed{-8x^2 + 14x - 6}$

Ex: $3x^2(-4x^3 - 12x + 8)$

$= 3x^2(-4x^3) + 3x^2(-12x) + 3x^2(8)$

$= \boxed{-12x^5 - 36x^3 + 24x^2}$

Dividing a polynomial by a monomial:

Ex: Divide. $\dfrac{-27x^4 + 15x^3 + 10x^2 - 3x}{3x}$

Note: same as $(-27x^4 + 15x^3 + 10x^2 - 3x) \div (3x)$

Note. $\dfrac{3}{7} + \dfrac{2}{7} = \dfrac{3+2}{7} = \dfrac{5}{7}$

Split into separate fractions:

$$\frac{-27x^4 + 15x^3 + 10x^2 - 3x}{3x} = \frac{-\cancel{27}^{9}x^4}{\cancel{3}_{1}x} + \frac{\cancel{15}^{5}x^3}{\cancel{3}_{1}x} + \frac{10x^2}{3x} - \frac{\cancel{3x}^{1}}{\cancel{3x}_{1}}$$

$$= -\frac{9x^3}{1} + \frac{5x^2}{1} + \frac{10x}{3} - 1$$

$$= \boxed{-9x^3 + 5x^2 + \frac{10}{3}x - 1}$$

Note: $\dfrac{10x}{3} = \dfrac{10}{3}x$ Why? $\dfrac{10}{3}x = \dfrac{10}{3} \cdot \dfrac{x}{1} = \dfrac{10x}{3}$

Careful: Never write $\frac{10}{3x}$ or $10/3x$

Carry out the multiplication or division.

Try these:

(1) $-4x^3(6-12x^4+2x)$

(2) $-3x^2y\,(2x^2y-5xy+4y^3)$

(3) $\dfrac{6a^4+2a^3-3a^2+5a-2}{2a^2}$

(4) $\dfrac{3a^4b^2-a^2b^2+a^2-6b^3}{4ab}$

(Done in class)

Multiplying Polynomials

Ex: $(\underbrace{3x}_{a}+\underbrace{5}_{b})\underbrace{(2x+1)}_{c}$

$= 3x(2x+1)+5(2x+1)$

$= 6x^2+3x+10x+5$

$= \boxed{6x^2+13x+5}$

Recall: distributive property

$a(b+c)=ab+ac$

$(a+b)\cdot c=ac+bc$

Example: Multiply.

$(3x-7)(4x+2)$

$= 3x(4x+2)-7(4x+2)$

$= 12x^2+6x-28x-14$

$= \boxed{12x^2-22x-14}$

Ex: $(2x^2-6)(5x^2-7x-4)$

$= 2x^2(5x^2-7x-4)-6(5x^2-7x-4)$

$= 10x^4-14x^3-8x^2-30x^2+42x+24$

$= \boxed{10x^4-14x^3-38x^2+42x+24}$

The "FOIL" Method: For multiplying 2 binomials

F: First
O: Outer
I: Inner
L: Last

Ex: $(3x-5)(4x-8)$

$= 12x^2 - 24x - 20x + 40$

$= \boxed{12x^2 - 44x + 40}$

You try some:

Ex 1: $(-2x+3)(4x+6)$

$= -8x^2 - 12x + 12x + 18$

$= \boxed{-8x^2 + 18}$

Ex 3: $(2x-5)^2$

$= (2x-5)(2x-5)$

$= 4x^2 - 10x - 10x + 25$

$= \boxed{4x^2 - 20x + 25}$

Ex: $(2x^2+4x-5)(x^2-6x-1)$

$= 2x^2(x^2-6x-1) + 4x(x^2-6x-1) - 5(x^2-6x-1)$

$= 2x^4 - 12x^3 - 2x^2$
$\quad + 4x^3 - 24x^2 - 4x$
$\quad - 5x^2 + 30x + 5$

$= \boxed{2x^4 - 8x^3 - 31x^2 + 26x + 5}$

Difference of 2 Squares Pattern:

$$(a+b)(a-b) = a^2 - b^2$$

$a^2 - \cancel{ab} + \cancel{ab} - b^2$

$= a^2 - b^2$

Ex 2: $(x-9)(x+9)$

$= x^2 + 9x - 9x - 81$

$= \boxed{x^2 - 81}$ This is a "difference of 2 squares"

Ex 4: $4x(-x^2+1)(3x-6)$

$= (-4x^3+4x)(3x-6)$

$= \boxed{-12x^4 + 24x^3 + 12x^2 - 24x}$

OR: $4x(-3x^3 + 6x^2 + 3x - 6)$

$= \boxed{-12x^4 + 24x^3 + 12x^2 - 24x}$

(same answer both ways)

Ex: $(x-2)(x+3)(x-4)$

$= \underbrace{((x-2)(x+3))}_{\text{Doing these 2 first}}(x-4)$

$= (x^2+3x-2x-6)(x-4)$

$= (x^2+x-6)(x-4)$

$= x^2(x-4) + x(x-4) - 6(x-4)$

$= x^3 - 4x^2$
$\quad + x^2 - 4x$
$\quad - 6x + 24$

$= \boxed{x^3 - 3x^2 - 10x + 24}$