1314-1-6-Notes-Other-Eqns

Wednesday, August 28, 2019 8:27 PM

1314-1-6-Notes-Other-Eqns

1.6: Other Types of Equations

Solving polynomial equations by factoring:

Example 1: Solve $9x^2 + 20x = -x^3$.

$x^3 + 9x^2 + 20x = 0$

$x(x^2 + 9x + 20) = 0$

$x(x+4)(x+5) = 0$

$x = 0 \mid x+4=0,\ x=-4 \mid x+5=0,\ x=-5$

This is not a quadratic eqn.

This is a cubic eqn.

Sol'n Set: $\boxed{\{0, -4, -5\}}$

Example 2: Solve $x^3 + 5x^2 - 9x = 45$.

$x^3 + 5x^2 - 9x - 45 = 0$

$(x^3 + 5x^2) + (-9x - 45) = 0$

$x^2(x+5) - 9(x+5) = 0$

$(x+5)(x^2-9) = 0$

$(x+5)(x+3)(x-3) = 0 \implies x = -5, -3, 3$

Sol'n set: $\boxed{\{-5, \pm 3\}}$

Example 3: Solve $x^3 = -16x$.

$x^3 = -16x$

$x^3 + 16x = 0$

$x(x^2 + 16) = 0$

$x = 0 \mid x^2 + 16 = 0$

$x^2 = -16$

$\sqrt{x^2} = \pm\sqrt{-16}$

$x = \pm\sqrt{-16}$

$x = \pm 4i$

Sol'n Set: $\boxed{\{0, \pm 4i\}}$

Example 4: Solve $2x^4 = 32x^2$.

Example 5: Solve $x^5 = x$

$$x^5 - x = 0$$
$$x(x^4 - 1) = 0$$
$$x(x^2+1)(x^2-1) = 0$$

completely factored over real number

$$\rightarrow x(x^2+1)(x+1)(x-1) = 0$$

$x = 0$ | $x^2+1=0$, $x^2 = -1$, $x = \pm\sqrt{-1}$, $x \pm i$ | $x+1=0$, $x=-1$ | $x-1=0$, $x=1$

Sol'n Set: $\boxed{\{0, \pm i, \pm 1\}}$

Solving equations involving radicals:

A radical equation is one in which the variable appears in a root, or radical sign. It may be a square root, cube root, or higher root. To solve these, we must isolate the radical and then raise both sides to a power.

Example 6: Solve $\sqrt{x} = 4$.

$$(\sqrt{x})^2 = (4)^2$$
$$x = 16$$

Sol'n Set: $\boxed{\{16\}}$

Check: $\sqrt{x} = 4$

$\sqrt{16} = 4$ ✓

Example 7: Solve $\sqrt[5]{x-7} = -2$

$$\sqrt[5]{x-7} = -2$$
$$\left(\sqrt[5]{x-7}\right)^5 = (-2)^5$$
$$x - 7 = -32$$
$$+7 \quad\quad +7$$
$$x = -25$$

Sol'n Set: $\boxed{\{-25\}}$

98
$2 \cdot 49$
$2 \cdot 7 \cdot 7$
$\underbrace{2 \cdot 7} \cdot 7$
$14 \cdot 7$

Example 8: Solve $\sqrt{2-x}-10=x$.

$\sqrt{2-x} - 10 = x$ Isolate the square root.

$+10 \quad +10$ (add 10 to both sides)

$\sqrt{2-x} = x+10$

$(\sqrt{2-x})^2 = (x+10)^2$ ← Square both sides.

$2-x = x^2+20x+100$

$-2 \quad +x \qquad +x \quad -2$

$0 = x^2+21x+98$

$0 = (x+14)(x+7)$

$x = -14, -7$

Let's check our solutions:

$x=-14$: $\sqrt{2-x} - 10 = x$

$\sqrt{2-(-14)} - 10 = -14$

$\sqrt{2+14} - 10 = -4$

$\sqrt{16} - 10 = -4$

$4-10 = -4$

FALSE! throw out -14 → $-6 = -4$

$x=-7$: $\sqrt{2-x} - 10 = x$

$\sqrt{2-(-7)} - 10 = -7$

$\sqrt{2+7} - 10 = -7$

$\sqrt{9} - 10 = -7$

$3-10 = -7$

$-7 = -7$ ✓ ok!

$\boxed{\text{Solution Set: } \{-7\}}$

> Extraneous Solutions:
> Raising both sides of an equation to an even power can introduce *extraneous solutions*, or *extraneous roots*, that are not solutions to the original equation.
>
> **Important**: Whenever you square both sides, or raise both sides to any even power, you must check your solutions in the original equation.

An extraneous solution (not a solution at all) is a value that is generated by a correct solution process, but does not check when substituted into the original equation.

Example 9: Solve $x-\sqrt{3-x}=-3$.

Example 10: Solve $\sqrt{1-2x} = 4 - \sqrt{x+5}$.

Isolate the most complicated radical.

$$\sqrt{1-2x} = 4-\sqrt{x+5}$$
$$(\sqrt{1-2x})^2 = (4-\sqrt{x+5})^2$$
$$= (4-\sqrt{x+5})(4-\sqrt{x+5})$$
$$1-2x = 16 - 4\sqrt{x+5} - 4\sqrt{x+5} + (\sqrt{x+5})^2$$
$$1-2x = 16 - 8\sqrt{x+5} + x+5$$

Isolate the remaining square root

$$1-2x = -8\sqrt{x+5} + x + 21$$
$$-21 \quad -x \qquad\qquad -x \quad -21$$
$$-20-3x = -8\sqrt{x+5}$$

Can multiply both sides by -1:

$$-1(-20-3x) = -(-8\sqrt{x+5})$$
$$20+3x = 8\sqrt{x+5}$$
$$3x+20 = 8\sqrt{x+5}$$
$$(3x+20)^2 = (8\sqrt{x+5})^2$$
$$9x^2+120x+400 = 64(x+5)$$
$$9x^2+120x+400 = 64x+320$$
$$-64x \quad -320 \qquad -64x \quad -320$$
$$9x^2+56x+80 = 0$$

$9x^2$, 80; 20x; $9x \cdot x$, $3x \cdot 3x$; $1\cdot 80$, $2\cdot 40$, $4\cdot 20$, $5\cdot 16$, $8\cdot 10$; 36x

then finish by factoring or quadratic formula

$$(9x+20)(x+4)=0$$

$9x+20=0$	$x+4=0$
$9x=-20$	$x=-4$
$x=-\frac{20}{9}$	

Check your solutions:

$x=-4$:
$$\sqrt{1-2x} = 4-\sqrt{x+5}$$
$$\sqrt{1-2(-4)} = 4-\sqrt{-4+5}$$
$$\sqrt{1+8} = 4-\sqrt{1}$$
$$\sqrt{9} = 4-1$$
$$3=3 \quad \text{True!!}$$

$x=-\frac{20}{9}$:
$$\sqrt{1-2x} = 4-\sqrt{x+5}$$
$$\sqrt{1-2(-\tfrac{20}{9})} = 4-\sqrt{-\tfrac{20}{9}+5}$$
$$\sqrt{1+\tfrac{40}{9}} = 4-\sqrt{-\tfrac{20}{9}+\tfrac{45}{9}}$$
$$\sqrt{\tfrac{49}{9}} = 4-\sqrt{\tfrac{25}{9}}$$
$$\tfrac{7}{3} = 4-\tfrac{5}{3}$$
$$\tfrac{7}{3} = \tfrac{12}{3}-\tfrac{5}{3}$$
$$\tfrac{7}{3} = \tfrac{7}{3} \ \checkmark \text{True! Whew!}$$

$$\boxed{\text{Solution Set: } \{-4, -\tfrac{20}{9}\}}$$

Solving equations with rational exponents:

Example 11: Solve $x^{2/3} + 4 = 9$.

$$x^{\frac{2}{3}} + 4 = 9$$
$$-4 \quad -4$$
$$x^{\frac{2}{3}} = 5$$
$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (5)^{\frac{3}{2}}$$
$$x = 5^{\frac{3}{2}}$$

Note: raising to $\frac{3}{2}$ power is square-roots and cubing $A^{\frac{3}{2}} = (A^{\frac{1}{2}})^3$

$$x = (5^3)^{1/2}$$
$$= \sqrt{125}$$
$$= \sqrt{25 \cdot 5}$$
$$= 5\sqrt{5}$$

$\boxed{\text{Sol'n Set: } \{5\sqrt{5}\}}$

Example 12: Solve $2x^{3/2} = 54$.

$$2x^{\frac{3}{2}} = 54$$
$$\frac{2x^{\frac{3}{2}}}{2} = \frac{54}{2}$$
$$x^{\frac{3}{2}} = 27$$
$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = 27^{\frac{2}{3}}$$
$$x = (27^{\frac{1}{3}})^2 = (\sqrt[3]{27})^2$$
$$= 3^2 = 9$$

$\boxed{\text{Sol'n Set } \{9\}}$

Solving equations quadratic in form:

An equation is called *quadratic in form* if, through an appropriate substitution, it can be rewritten as a quadratic equation in another variable.

<u>**Example 13:**</u> Solve $x^4 - 7x^2 - 8 = 0$.

want: $au^2 + bu + c = 0$

$(x^2)^2 - 7x^2 - 8 = 0$

could let $u = x^2$ and write $u^2 - 7u - 8 = 0$

$(u-8)(u+1) = 0$

$u - 8 = 0 \mid u + 1 = 0$

$u = 8 \mid u = -1$

$x^2 = 8 \mid x^2 = -1$

$x = \pm\sqrt{8}$, $x = \pm\sqrt{-1}$

$x = \pm 2\sqrt{2}$, $x = \pm i$

$\boxed{\text{Sol'n Set: } \{\pm 2\sqrt{2}, \pm i\}}$

<u>**Example 14:**</u> Solve $2x^6 - x^3 = 3$.

$2x^6 - x^3 - 3 = 0$

$2\underbrace{(x^3)^2}_{u} - \underbrace{(x^3)}_{u} - 3 = 0$

could write $2u^2 - u - 3 = 0$ with $u = x^3$

$(2u - 3)(u + 1) = 0$

$2u - 3 = 0 \mid u = -1$

$2u = 3 \mid x^3 = -1$

$u = \frac{3}{2} \mid \sqrt[3]{x^3} \quad \sqrt[3]{-1}$

$x^3 = \frac{3}{2} \mid x = \sqrt[3]{-1}$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{3}{2}} \mid x = -1$

$x = \sqrt[3]{\frac{3}{2}}$

$(2x^3 - 3)(x^3 + 1) = 0$

$2x^3 - 3 = 0 \mid x^3 + 1 = 0$

$2x^3 = 3 \mid x^3 = -1$

$x^3 = \frac{3}{2}$

(take cube root of both sides)

$(?)^3 = -1$

answer: -1

so $\sqrt[3]{-1} = -1$

Rationalize denominator:

$\sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$

$= \frac{\sqrt[3]{3}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$= \frac{\sqrt[3]{12}}{2}$

$\boxed{\text{Sol'n Set: } \left\{\sqrt[3]{\frac{3}{2}}, -1\right\}}$

$\boxed{\text{Or } \left\{\frac{\sqrt[3]{12}}{2}, -1\right\}}$

<u>**Example 15:**</u> Solve $x^{-2} + 5x^{-1} + 6 = 0$.

$x^{-2} + 5x^{-1} + 6 = 0$

$\underbrace{(x^{-1})^2}_{u} + 5\underbrace{(x^{-1})}_{u} + 6 = 0$

$u = x^{-1} \Rightarrow \quad u^2 + 5u + 6 = 0$

$(u + 2)(u + 3) = 0$

$u = -2 \mid u = -3$

$x^{-1} = -2 \mid x^{-1} = -3$

$\frac{1}{x} = -2 \mid \frac{1}{x} = -3$

$\frac{1}{x} = -\frac{2}{1}$

$1 = -2x$

$-\frac{1}{2} = x$

$\boxed{\text{Sol'n Set: } \left\{-\frac{1}{2}, -\frac{1}{3}\right\}}$

$$x^{\frac{2}{3}} - 5x^{\frac{1}{3}} = 36$$

Example 16: Solve $x^{2/3} - 5x^{1/3} = 36$.

$$\left(x^{\frac{1}{3}}\right)^2 - 5x^{\frac{1}{3}} - 36 = 0$$

$$u = x^{\frac{1}{3}} \Rightarrow u^2 - 5u - 36 = 0$$

$$(u-9)(u+4) = 0$$

$$u = 9, \quad u = -4$$

$$x^{\frac{1}{3}} = 9, \quad x^{\frac{1}{3}} = -4$$

$$\sqrt[3]{x} = 9 \qquad \sqrt[3]{x} = -4$$

$$\left(\sqrt[3]{x}\right)^3 = (9)^3 \qquad \left(\sqrt[3]{x}\right)^3 = (-4)^3$$

$$x = 729 \qquad x = -64$$

Sol'n Set.

$$\boxed{\{729, -64\}}$$

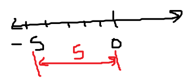
Solving absolute value equations:

Absolute value is another way of saying "size" or "magnitude" or "distance from zero".

Note: $|3| = 3$

$|-3| = 3$

For example, $|-5|$ is the distance from -5 to 0 on the number line.
$|x|$ is the distance from x to 0 on the number line.
So if we say that $|x| = 8$, then it must be that $x = 8$ or $x = -8$.



To solve absolute value equations:

- Isolate the absolute value on one side.
- If C is positive, use the fact that $|x| = C$ means that $x = \pm C$ to write two new equations.
- Solve each of these resulting equations.

Example 17: Solve $|2x-3| - 7 = 0$.

Isolate the absolute value: $|2x-3| = 7$

$$2x - 3 = -7 \quad \text{or} \quad 2x - 3 = 7$$

$$+3 \quad +3 \qquad\qquad +3 \quad +3$$

$$2x = -4 \qquad\qquad 2x = 10$$

$$\frac{2x}{2} = \frac{-4}{2} \qquad\qquad \frac{2x}{2} = \frac{10}{2}$$

$$x = -2 \qquad\qquad x = 5$$

Sol'n Set: $\boxed{\{-2, 5\}}$

Check:

$x = -2$

$$|2x-3| - 7 = 0$$

$$|2(-2)-3| - 7 = 0$$

$$|-4-3| - 7 = 0$$

$$|-7| - 7 = 0$$

$$7 - 7 = 0 \quad 0$$

$$= 0 \checkmark$$

$x = 5$

$$|2(5)-3| - 7 = 0$$

$$|10-3| - 7 = 0$$

$$|7| - 7 = 0$$

$$7 - 7 = 0$$

$$0 = 0 \checkmark$$

Important: $|2x-3| \neq 2x+3$ $|x+5| \neq x+5$ not true in general!

$|x-5| \neq x+5$ $|5-2x| \neq 5+2x$

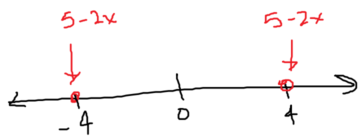
1.6.1

Example 18: Solve $2|5-2x|+6=14$.

$-6 \quad -6$

$2|5-2x| = 8$

Divide by 2: $|5-2x| = 4$



$5-2x = -4$ OR $5-2x = 4$

$-5 \quad -5 \qquad -5 \quad -5$

$-2x = -9 \qquad -2x = -1$

$x = \frac{-9}{-2} = \frac{9}{2} \qquad x = \frac{-1}{-2} = \frac{1}{2}$

Sol'n Set: $\boxed{\left\{\frac{9}{2}, \frac{1}{2}\right\}}$

Example 19: Solve $20+|2x+4|=15$.

$-20 \quad -20$

$|2x+4| = -5$

absolute value of some number is -5 $\Rightarrow$ IMPOSSIBLE!

$\boxed{\text{No Solution}}$

Sol'n Set is $\emptyset$

$\emptyset$ means "the empty set"

Example 20: Solve $|x+3|=|2x+1|$.

"abs. value of some number is equal to abs value of some other number"

How can two numbers have the same absolute value?

Either both the numbers are the same, or the two numbers are opposites.

$x+3 = 2x+1$ OR $x+3 = -(2x+1)$

$-2x \quad -2x \qquad x+3 = -2x-1$

$-x+3 = 1 \qquad +2x \quad +2x$

$-3 \quad -3 \qquad 3x+3 = -1$

$-x = -2 \qquad -3 \quad -3$

$x = 2 \qquad 3x = -4$

$x = -\frac{4}{3}$

$\boxed{\left\{2, -\frac{4}{3}\right\}}$

Example 21: Solve $\frac{1}{|x+3|} = 2$.