

1.6: Other Types of Equations

Solving polynomial equations by factoring:

Example 1: Solve $9x^2 + 20x = -x^3$.

Example 2: Solve $x^3 + 5x^2 - 9x = 45$.

Example 3: Solve $x^3 = 16x$.

Example 4: Solve $2x^4 = 32x^2$.

Example 5: Solve $x^5 = x$

Solving equations involving radicals:

A radical equation is one in which the variable appears in a root, or radical sign. It may be a square root, cube root, or higher root. To solve these, we must isolate the radical and then raise both sides to a power.

Example 6: Solve $\sqrt{x} = 4$.

Example 7: Solve $\sqrt[5]{x-7} = -2$

Example 8: Solve $\sqrt{2-x} - 10 = x$.

Extraneous Solutions:

Raising both sides of an equation to an even power can introduce *extraneous solutions*, or *extraneous roots*, that are not solutions to the original equation.

Important: Whenever you square both sides, or raise both sides to any even power, you must check your solutions in the original equation.

Example 9: Solve $x - \sqrt{3-x} = -3$.

Example 10: Solve $\sqrt{1-2x} = 4 - \sqrt{x+5}$.

Solving equations with rational exponents:

Example 11: Solve $x^{\frac{2}{3}} + 4 = 9$.

Example 12: Solve $2x^{\frac{3}{2}} = 54$.

Solving equations quadratic in form:

An equation is called *quadratic in form* if, through an appropriate substitution, it can be rewritten as a quadratic equation in another variable.

Example 13: Solve $x^4 - 7x^2 - 8 = 0$.

Example 14: Solve $2x^6 - x^3 = 3$.

Example 15: Solve $x^{-2} + 5x^{-1} + 6 = 0$.

Example 16: Solve $x^{2/3} - 5x^{1/3} = 36$.

Solving absolute value equations:

Absolute value is another way of saying *Size* or *magnitude* or *distance from zero*.

For example, $|-5|$ is the distance from -5 to 0 on the number line.

$|x|$ is the distance from x to 0 on the number line.

So if we say that $|x| = 8$, then it must be that $x = 8$ or $x = -8$.

To solve absolute value equations:

- Isolate the absolute value on one side.
- If C is positive, use the fact that $|x| = C$ means that $x = \pm C$ to write two new equations.
- Solve each of these resulting equations.

Example 17: Solve $|2x - 3| - 7 = 0$.

Example 18: Solve $2|5 - 2x| + 6 = 14$.

Example 19: Solve $20 + |2x + 4| = 15$.

Example 20: Solve $|x + 3| = |2x + 1|$.

Example 21: Solve $\frac{1}{|x + 3|} = 2$.