1.6 #11 Solve.

$$\sqrt{3x+18} = x$$

$$\left(\sqrt{3x+18}\right)^2 = (x)^2$$

$$3x+18 = x^2$$

$$0 = x^2 - 3x - 18$$

$$0 = (x-6)(x+3)$$

$$x=6, \quad x=-3$$

We must check our answers

$x=6$

$$\sqrt{3x+18} = x$$

$$\sqrt{3(6)+18} = 6$$

$$\sqrt{18+18} = 6$$

$$\sqrt{36} = 6$$

$$6 = 6$$ ✓ OK!

Sol'n Set $\boxed{\{6\}}$

$x=-3$

$$\sqrt{3(-3)+18} = -3$$

$$\sqrt{-9+18} = -3$$

$$\sqrt{9} = -3$$

$$3 = -3$$

False!

Throw out $x=-3$

## 1.7: Linear Inequalities

An *inequality* is similar to an equation, except that in place of the equal sign, an inequality will have one of the symbols $<, >, \leq, \geq$. An inequality is called *linear* if each term is a constant or a multiple of the variable (i.e. it has no $x^2, \frac{1}{x}, \sqrt{x}$, etc.).

Ex:

Inequalities using the symbols <, > are called *strict* inequalities.

To *solve* an equation or inequality means to find all the values of the variable that make the equation or inequality true. Most of the equations we will solve have 1, 2, or maybe several solutions. Most inequalities have infinitely many solutions. Usually the solutions to an inequality form an interval or a union of intervals on the number line. We=ll only concern ourselves with real solutions to inequalities. (No complex numbers!)

Just as with a linear equation, we'll solve a linear inequality by transforming the inequality into a series of *equivalent* inequalities by adding, multiplying, etc. the same thing to both sides.

In the following rules, the symbol $\Leftrightarrow$ means "is equivalent to".

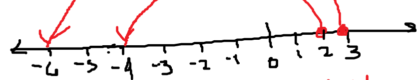
**Rules for inequalities:** (these rules also apply to strict inequalities)

| | |
|---|---|
| 1. $A \leq B \quad \Leftrightarrow \quad A + C \leq B + C$. | **Adding** the same quantity to both sides does not change the inequality. |
| 2. $A \leq B \quad \Leftrightarrow \quad A - C \leq B - C$. | **Subtracting** the same quantity from both sides does not change the inequality. |
| 3. If $C > 0$, then $A \leq B \quad \Leftrightarrow \quad CA \leq CB$. | **Multiplying** both sides by the same **positive** quantity does not change the inequality. |
| 4. If $C < 0$, then $A \leq B \quad \Leftrightarrow \quad CA \geq CB$. | **Multiplying** both sides by the same **negative** quantity **reverses** the inequality. |
| 5. If $A > 0$ and $B > 0$, then $A \leq B \quad \Leftrightarrow \quad \frac{1}{A} \geq \frac{1}{B}$. $2 < 3$ $\frac{1}{2} > \frac{1}{3}$ | Taking reciprocals of each side of an inequality involving **positive** quantities **reverses** the inequality. |
| 6. If $A \leq B$ and $C \leq D$, then $A + C \leq B + D$. | Inequalities can be added. |
| 7. If $A > 0$, then $\frac{1}{A} > 0$. If $A < 0$, then $\frac{1}{A} < 0$. | Taking reciprocals does not change the sign of a quantity. |

Why must we reverse the inequality sign when multiplying/dividing by a negative? 1.7.2

**Example using numbers:**

2 < 3 true




multiply both sides by −2: −2(2) < −2(3)

−4 < −6 False!

Multiplying or dividing by a negative swaps the order on the number line. So we must reverse the inequality signs to keep it true.

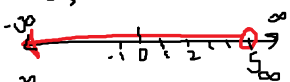
**Interval notation:**

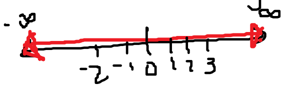

$(-3,5)$ All $x$ such that $-3 < x < 5$ $\{x \mid -3 < x < 5\}$


$[-3,5]$ All $x$ such that $-3 \le x \le 5$ $\{x \mid -3 \le x \le 5\}$


$[-3,5)$ All $x$ such that $-3 \le x < 5$ $\{x \mid -3 \le x < 5\}$


$[-3,\infty)$ All $x$ such that $x \ge -3$ $\{x \mid x \ge -3\}$


$(-\infty,5)$ All $x$ such that $x < 5$ $\{x \mid x < 5\}$


$(-\infty,\infty)$ All real numbers alternative notation for all real numbers: $\mathbb{R}$

When solving inequalities, you should be able to write all answers in interval notation.

<u>**Example 1:**</u> Solve $4x+1 \le 7$.

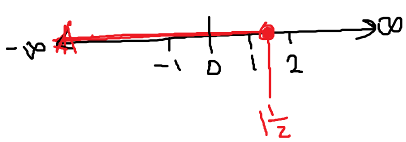
$-1 \quad -1$

$4x \le 6$

$\frac{4x}{4} \le \frac{6}{4}$

$x \le \frac{3}{2}$

$x \le 1\frac{1}{2}$




Sol'n set: $\boxed{\left(-\infty, \frac{3}{2}\right]}$ or $\boxed{\left(-\infty, 1\frac{1}{2}\right]}$

<u>**Example 2:**</u> Solve $-2(6x+1) > 22$.

$-12x - 2 > 22$

$+2 \quad +2$

$-12x > 24$

$\frac{-12x}{-12} < \frac{24}{-12}$ [Reverse the inequality sign!!]

$x < -2$


Sol'n Set: $\boxed{(-\infty, -2)}$

**Example 3:** Solve $\frac{1}{5-2x} < 0$.

If two numbers are reciprocals they have the same + or − sign.
(Either they are both positive or both negative).
Since $\frac{1}{5-2x}$ is negative, it must be that $5-2x$ is negative also.

$5-2x<0$

$-2x<-5$

$\frac{-2x}{-2} > \frac{-5}{-2}$

$x > \frac{5}{2}$

$x > 2\frac{1}{2}$

OR

$5-2x<0$

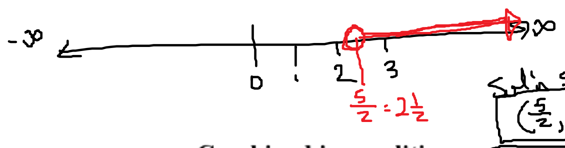
$5<2x$

$\frac{5}{2} < \frac{2x}{2}$

$\frac{5}{2} < x$

$x > \frac{5}{2}$ → $\{x \mid x > \frac{5}{2}\}$

[Rewrite with variable on the left!]



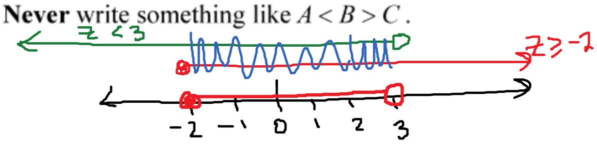

Sol'n set: $\boxed{(\frac{5}{2}, \infty)}$ OR $\boxed{(2\frac{1}{2}, \infty)}$

**Combined inequalities:**

Sometimes, two inequalities can be written in a more compact way. This is called a *combined inequality* or a pair of *simultaneous inequalities*.

If you see $A < B \le C$, this means $A < B$ **and** $B \le C$. To combine two inequalities this way, both inequality signs must be going the same direction. **Never** write something like $A < B > C$.

**Example 4:** Solve $-3 \le 2z+1 < 7$.




This means: $-3 \le 2z+1$ and $2z+1<7$

$-1 \quad -1 \qquad\qquad -1 \quad -1$

$-4 \le 2z \qquad\qquad 2z < 6$

$\frac{-4}{2} \le \frac{2z}{2} \qquad\qquad \frac{2z}{2} < \frac{6}{2}$

$-2 \le z \qquad\qquad z < 3$

$z \ge -2$ and

Put back together: $-2 \le z < 3$

we want the values that satisfy both (so the overlapping section)

Sol'n set: $\boxed{[-2, 3)}$

or, do them simultaneously:

$-3 \le 2z+1 < 7$

$-1 \quad -1 \quad -1$

$-4 \le 2z < 6$

$\frac{-4}{2} \le \frac{2z}{2} < \frac{6}{2}$

$-2 \le z < 3$

$\boxed{\{z \mid -2 \le z < 3\}}$

**Example 5:** Solve $-\frac{2}{3} < \frac{9-6x}{2} \le \frac{1}{6}$.

Multiply both sides by 6 to clear the fractions:

$\frac{\cancel{6}^{2}}{1}\left(-\frac{2}{\cancel{3}_{1}}\right) < \frac{\cancel{6}^{3}}{1}\left(\frac{9-6x}{\cancel{2}_{1}}\right) \le \frac{\cancel{6}^{1}}{1}\left(\frac{1}{\cancel{6}_{1}}\right)$

$-4 < 3(9-6x) \le 1$

$-4 < 27 - 18x \le 1$

$-27 \quad -27 \qquad -27$

$-31 < -18x \le -26$

$\frac{-31}{-18} > \frac{-18x}{-18} \ge \frac{-26}{-18}$

$\frac{31}{18} > x \ge \frac{13}{9}$

Rewrite with signs pointing this way $\le <$

$\frac{13}{9} \le x < \frac{31}{18}$

$1\frac{4}{9} \le x < 1\frac{13}{18}$

$\boxed{\{x \mid 1\frac{8}{18} \le x < 1\frac{13}{18}\}}$

$\boxed{[1\frac{8}{18}, 1\frac{13}{18})}$

OR $\boxed{[1\frac{4}{9}, 1\frac{13}{18})}$

**<u>Example 6:</u>** Solve $2 < 8 - 3x < -3$.

This means $2 < 8 - 3x$ <u>and</u> $8 - 3x < -3$

$2 <$ some number <u>and</u> that same number $< -3$ ??? Impossible!!

$\boxed{\text{No solution}}$

**Absolute value inequalities:**

To solve these we must remember that absolute value means **distance from zero!!!!**

**<u>Example 1:</u>** Solve $|x| \le 5$.

**<u>Example 2:</u>** Solve $|x| > 6$.

**Summary:** For a *positive* number $c$:

$$|x| < c \quad \Leftrightarrow \quad -c < x < c$$
$$|x| \le c \quad \Leftrightarrow \quad -c \le x \le c$$
$$|x| > c \quad \Leftrightarrow \quad x < -c \text{ or } x > c$$
$$|x| \ge c \quad \Leftrightarrow \quad x \le -c \text{ or } x \ge c$$

**<u>Example 3:</u>** Solve $|-3x + 1| < 4$.

**Example 6:** Solve $2 < 8 - 3x < -3$.

$\cup$: union. A or B is true
$\cap$: intersection A and B is true

**Absolute value inequalities:**

To solve these we must remember that absolute value means **distance from zero!!!!**

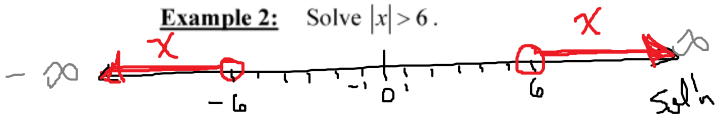
Always draw a number line for these.

**Example 1:** Solve $|x| \le 5$.


Sol'n set: $\{x \mid -5 \le x \le 5\}$

$[-5, 5]$

**Example 2:** Solve $|x| > 6$.




Sol'n Set: $\{x \mid x < -6 \text{ or } x > 6\}$

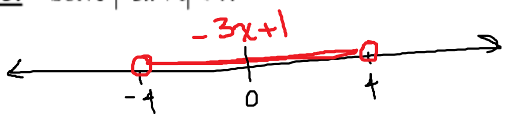
Interval Notation: $(-\infty, -6) \cup (6, \infty)$

**Summary:** For a *positive* number *c*:

$$|x| < c \quad \Leftrightarrow \quad -c < x < c$$
$$|x| \le c \quad \Leftrightarrow \quad -c \le x \le c$$
$$|x| > c \quad \Leftrightarrow \quad x < -c \text{ or } x > c$$
$$|x| \ge c \quad \Leftrightarrow \quad x \le -c \text{ or } x \ge c$$

**Example 3:** Solve $|-3x+1| < 4$.

Equivalent to $|y| < 4$ with $y = -3x+1$




$-4 < -3x+1 < 4$
$\quad -1 \quad\quad -1 \quad -1$

$-5 < -3x < 3$

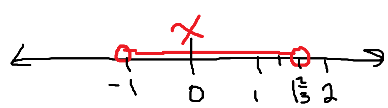
Reverse the inequality signs (divided by a negative)

$\frac{-5}{-3} > \frac{-3x}{-3} > \frac{3}{-3}$

$\frac{5}{3} > x > -1$

Rewrite with signs pointing this way $<\,<$ $\rightarrow$ $-1 < x < \frac{5}{3}$

$-1 < x < 1\frac{2}{3}$




Sol'n Set: $(-1, 1\frac{2}{3})$ interval notation

$\{x \mid -1 < x < 1\frac{2}{3}\}$ set roster notation

or $\{x \mid -1 < x < \frac{5}{3}\}$

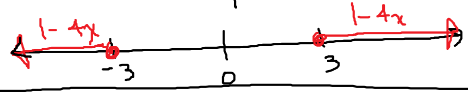
**Example 4:** Solve $2|1-4x|+1 \ge 7$.

Isolate the absolute value:

$$2\cdot|1-4x| \ge 6$$

$$\frac{\cancel{2}|1-4x|}{\cancel{2}} \ge \frac{6}{2}$$

$$|1-4x| \ge 3$$



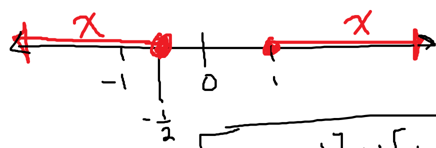

$1-4x \le -3$ OR $1-4x \ge 3$

$-1 \quad -1 \qquad -1 \quad -1$

$-4x \le -4 \qquad -4x \ge 2$

$\frac{-4x}{-4} \ge \frac{-4}{-4} \qquad \frac{-4x}{-4} \le \frac{2}{-4}$

$x \ge 1$ OR $x \le -\frac{1}{2}$




Sol'n Set: $\boxed{(-\infty, -\tfrac{1}{2}] \cup [1, \infty)}$

**Example 5:** $13-3|4x-5| \le 1$

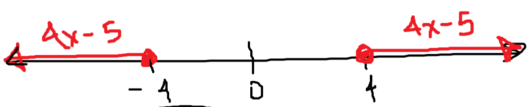
$-13 \qquad -13$

Isolate the absolute value:

$$-3\cdot|4x-5| \le -12$$

$$\frac{-\cancel{3}\cdot|4x-5|}{-\cancel{3}} \ge \frac{-12}{-3}$$

$$|4x-5| \ge 4$$



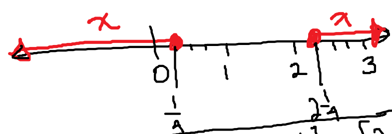

$4x-5 \le -4$ or $4x-5 \ge 4$

$+5 \quad +5 \qquad +5 \quad +5$

$4x \le 1$ OR $4x \ge 9$

$\frac{4x}{4} \le \frac{1}{4} \qquad \frac{4x}{4} \ge \frac{9}{4}$

$x \le \frac{1}{4}$ OR $x \ge 2\frac{1}{4}$




Sol'n Set: $\boxed{(-\infty, \tfrac{1}{4}] \cup [2\tfrac{1}{4}, \infty)}$

**Example 6:** Solve $7+|2+3x| \le 4$.

$-7 \qquad -7$

Isolate the abs. value: $|2+3x| \le -3$

Not possible! Absolute value is always at least 0

$\boxed{\text{No solution}}$

**Example 7:** Solve $|9x+2| \ge -5$.

Always true!!

Sol'n Set is $\boxed{\text{all real numbers}}$

$\boxed{(-\infty, \infty)}$ in interval notation