

1.7: Linear Inequalities

An *inequality* is similar to an equation, except that in place of the equal sign, an inequality will have one of the symbols $<, >, \leq, \geq$. An inequality is called *linear* if each term is a constant or a multiple of the variable (i.e. it has no $x^2, \frac{1}{x}, \sqrt{x}$, etc.).

Ex:

Inequalities using the symbols $<, >$ are called *strict* inequalities.

To *solve* an equation or inequality means to find all the values of the variable that make the equation or inequality true. Most of the equations we will solve have 1, 2, or maybe several solutions. Most inequalities have infinitely many solutions. Usually the solutions to an inequality form an interval or a union of intervals on the number line. We'll only concern ourselves with real solutions to inequalities. (No complex numbers!)

Just as with a linear equation, we'll solve a linear inequality by transforming the inequality into a series of *equivalent* inequalities by adding, multiplying, etc. the same thing to both sides.

In the following rules, the symbol $\Leftrightarrow$ means A is equivalent to B.

Rules for inequalities: (these rules also apply to strict inequalities)

1. $A \leq B \Leftrightarrow A + C \leq B + C$.	Adding the same quantity to both sides does not change the inequality.
2. $A \leq B \Leftrightarrow A - C \leq B - C$.	Subtracting the same quantity from both sides does not change the inequality.
3. If $C > 0$, then $A \leq B \Leftrightarrow CA \leq CB$.	Multiplying both sides by the same positive quantity does not change the inequality.
4. If $C < 0$, then $A \leq B \Leftrightarrow CA \geq CB$.	Multiplying both sides by the same negative quantity reverses the inequality.
5. If $A > 0$ and $B > 0$, then $A \leq B \Leftrightarrow \frac{1}{A} \geq \frac{1}{B}$.	Taking reciprocals of each side of an inequality involving positive quantities reverses the inequality.
6. If $A \leq B$ and $C \leq D$, then $A + C \leq B + D$.	Inequalities can be added.
7. If $A > 0$, then $\frac{1}{A} > 0$. If $A < 0$, then $\frac{1}{A} < 0$.	Taking reciprocals does not change the sign of a quantity.

Example using numbers:**Interval notation:**

$(-3, 5)$ All x such that $-3 < x < 5$

$[-3, 5]$ All x such that $-3 \leq x \leq 5$

$[-3, 5)$ All x such that $-3 \leq x < 5$

$[-3, \infty)$ All x such that $x \geq -3$

$(-\infty, 5)$ All x such that $x < 5$

$(-\infty, \infty)$ All real numbers

When solving inequalities, you should be able to write all answers in interval notation.

Example 1: Solve $4x + 1 \leq 7$.

Example 2: Solve $-2(6x + 1) > 22$.

Example 3: Solve $\frac{1}{5-2x} < 0$.

Combined inequalities:

Sometimes, two inequalities can be written in a more compact way. This is called a *combined inequality* or a pair of *simultaneous inequalities*.

If you see $A < B \leq C$, this means $A < B$ **and** $B \leq C$. To combine two inequalities this way, both inequality signs must be going the same direction. **Never** write something like $A < B > C$.

Example 4: Solve $-3 \leq 2z + 1 < 7$.

Example 5: Solve $-\frac{2}{3} < \frac{9-6x}{2} \leq \frac{1}{6}$.

Example 6: Solve $2 < 8 - 3x < -3$.

Absolute value inequalities:

To solve these we must remember that absolute value means **distance from zero!!!!**

Example 1: Solve $|x| \leq 5$.

Example 2: Solve $|x| > 6$.

Summary: For a *positive* number c :

$$|x| < c \Leftrightarrow -c < x < c$$

$$|x| \leq c \Leftrightarrow -c \leq x \leq c$$

$$|x| > c \Leftrightarrow x < -c \text{ or } x > c$$

$$|x| \geq c \Leftrightarrow x \leq -c \text{ or } x \geq c$$

Example 3: Solve $|-3x + 1| < 4$.

Example 4: Solve $2|1 - 4x| + 1 \geq 7$.

Example 5: $13 - 3|4x - 5| \leq 1$

Example 6: Solve $7 + |2 + 3x| \leq 4$.

Example 7: Solve $|9x + 2| \geq -5$.