Review Problems

① $3x^2 = x + 5$

$3x^2 - x - 5 = 0$

$$x = \frac{1 \pm \sqrt{1^2 - 4(3)(-5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{1 + 60}}{6} = \frac{1 \pm \sqrt{61}}{6}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sol'n Set: $\boxed{\left\{ \frac{1 \pm \sqrt{61}}{6} \right\}}$

② $|6 - 4x| = 2$

$\left| \text{unknown number} \right| = 2$

$6 - 4x = -2$ or $6 - 4x = 2$

$-4x = -8$

$\frac{-4x}{-4} = \frac{-8}{-4}$

$x = 2$

$-4x = -4$

$\frac{-4x}{-4} = \frac{-4}{-4}$

$x = 1$

unknown number $= -2$ or unknown number $= 2$

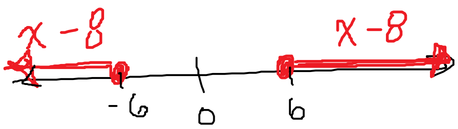
Sol'n Set: $\boxed{\{1, 2\}}$

(3) $2|x-8| \geq 12$

[Isolate the absolute value]

$|x-8| \geq 6$

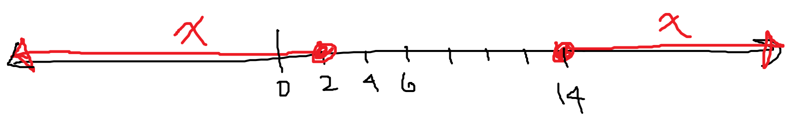
$|\text{unknown number}| \geq 6$



$x - 8 \leq -6$ OR $x - 8 \geq 6$

$+8 \quad +8 \qquad +8 \quad +8$

$x \leq 2$ OR $x \geq 14$



$\boxed{\text{Sol'n Set: } (-\infty, 2] \cup [14, \infty)}$

2.1: Basics of Functions

Definition: A *relation* is any set of ordered pairs. The set of all first components of the ordered pairs is called the *domain* of the relation, and the set of all second components is called the *range* of the relation.

Domain: Set of inputs
Range: Set of outputs

Example 1: $\{(1,5),(-3,6),(2,4),(1,6)\}$ What are the domain and range of this relation?

Domain: $\{1,-3,2\}$ Range: $\{5,6,4\}$

Definition: A *function* is a correspondence from a first set, called the domain, to a second set, called the range, such that each element in the domain corresponds to exactly one element in the range.

We can think of the domain as containing inputs and the range as containing outputs.

Definition: (informal) A *function* is a relation or rule in which every "input" is associated with exactly one "output".

Another way to think about it: A function is like a machine. It takes an x (the input) and spits out exactly one y (the output).

Example 2: Are the following relations functions?

a. $\{(1,5),(-3,6),(2,4),(1,6)\}$

Domain → Range
$1 \to 5$
$1 \to 6$
$-3 \to 6$
$2 \to 4$

Not a function

The input 1 is associated with 2 outputs

b. $\{(2,5),(-3,6),(3,4),(1,6)\}$

Domain → Range
$2 \to 5$
$-3 \to 6$
$1 \to 6$
$3 \to 4$

Yes, this is a function

each input goes to only 1 output

Function Notation

$f(Math) =$ ☺ [Read "f of $Math$ equals happy face"]

$Math$: the input (an element of the domain)

☺ : the output (an element of the range)

f : the name of the function (usually this is a letter, often f, g, or h.)

In algebra, lots of functions are named f, g, h

In trigonometry, there are functions named cos, sin

In algebra, we will see functions named ln $\log_{10}$

This notation is very helpful if we want our function to take real numbers as inputs. We can't list them all in a table. For a function named *f*, $f(x)$ represents the value of the function at the number *x*.

Example 1: Consider the relation $\{(3,5),(4,6),(-1,1),(1.5,3.5),(-7,-5)\}$. What equation describes the relationship?

$y = x + 2$

Notice that $y = 5$ when $x = 3$. Also $y = 6$ when $x = 4$. Writing $f(x)$ helps keep track of which *x* goes with which *y*.

$(3,5) \Rightarrow f(3) = 5$

$(4,6) \Rightarrow f(4) = 6$

$(-7,-5) \Rightarrow f(-7) = -5$

For $y = x + 2$, we can write

$f(x) = x + 2$

Ex: $f(3) = 3 + 2 = 5$

Determining whether an equation defines a function:

Example 3: Does the equation $x^2 + 5y = 7$ define *y* as a function of *x*?

Solve for y: $5y = 7 - x^2$

$\frac{5y}{5} = \frac{7 - x^2}{5}$

$y = \frac{7}{5} - \frac{x^2}{5}$

Just 1 answer for y so we could write $f(x) = \frac{7}{5} - \frac{x^2}{5}$

$\boxed{\text{Yes this is a function}}$

Example 4: Does the equation $x^2 - 3y^2 = 6$ define *y* as a function of *x*?

Solve for y: $-3y^2 = 6 - x^2$

$\frac{-3y^2}{-3} = \frac{6}{-3} - \frac{x^2}{-3}$

$y^2 = -2 + \frac{x^2}{3}$

$y = \pm\sqrt{-2 + \frac{x^2}{3}}$

2 answers for y, so $\boxed{\text{Not a function}}$

Example 5: Does the equation $x + y^3 = 7$ define *y* as a function of *x*?

Solve for y: $y^3 = 7 - x$

$y = \sqrt[3]{7 - x}$

Just 1 answer for y, so $\boxed{\text{yes, it is a function}}$

We use a $\pm$ when taking square root or any even root of both sides (when we square, we get the same thing when squaring a positive or negative)

When we take an odd root of a negative, get a negative

When we take an ~~odd~~ odd root of a positive, we get a positive

Even roots: need $\pm$

Odd roots: No $\pm$

Evaluating functions:

Example 2: Suppose $f(x) = 3x - 5$. Calculate $f(3)$.

substitute 3 for x

$f(3) = 3(3) - 5$
$= 9 - 5 = \boxed{4}$

Example 3: Suppose $g(x) = 3x^2 + 4x - 7$. Calculate $g(-2)$.

$g(\boxed{}) = 3\boxed{}^2 + 4\boxed{} - 7$

$g(\boxed{-2}) = 3\boxed{-2}^2 + 4\boxed{-2} - 7$

$g(-2) = 3(-2)^2 + 4(-2) - 7$
$= 3(4) - 8 - 7 = 12 - 15 = \boxed{-3}$

Example 4: Suppose $f(x) = -17$. Calculate $f(-2)$.

$f(\boxed{}) = -17$

$f(\boxed{-2}) = -17$ so $\boxed{f(-2) = -17}$

This is called a constant function. Every input gets sent to the same output, -17

Example 5: Suppose $f(x) = 4x - 8$.

$f(\boxed{}) = 4\boxed{} - 8$

- Calculate $f(7x - 2)$.

$f(\boxed{7x-2}) = 4\boxed{7x-2} - 8$

$f(7x-2) = 4(7x-2) - 8$
$= 28x - 8 - 8 = \boxed{28x - 16}$

- Calculate $f(a^2)$.

$f(\boxed{a^2}) = 4\boxed{a^2} - 8$

$\boxed{f(a^2) = 4a^2 - 8}$

Example 6: Suppose $g(x) = x^2 - 4x + 3$.

$g(\boxed{}) = \boxed{}^2 - 4\boxed{} + 3$

- Calculate $g(-x)$.

$g(\boxed{-x}) = \boxed{-x}^2 - 4\boxed{-x} + 3$

$g(-x) = (-x)^2 - 4(-x) + 3$

$\boxed{g(-x) = x^2 + 4x + 3}$

- Calculate $g(x - 2)$.

$g(x-2) = (x-2)^2 - 4(x-2) + 3$
$= (x-2)(x-2) - 4(x-2) + 3$
$= x^2 - 2x - 2x + 4 - 4x + 8 + 3$

$x^2 = 4 \quad x + 4 - 4x + 11$

$\boxed{g(x-2) = x^2 - 8x + 15}$

Example 7: Suppose $f(x) = \dfrac{2x-5}{7-x}$.

- Calculate $f(-3)$.

$$f(-3) = \frac{2(-3)-5}{7-(-3)} = \frac{-6-5}{7+3} = \frac{-11}{10}$$

$$\boxed{f(-3) = -\frac{11}{10}}$$

- Calculate $f(\frac{1}{3})$.

$$f\left(\frac{1}{3}\right) = \frac{2\left(\frac{1}{3}\right)-5}{7-\frac{1}{3}} = \frac{\frac{2}{3}-5}{7-\frac{1}{3}} \cdot \frac{3}{3} = \frac{2-15}{21-1} = \frac{-13}{20}$$

$$\boxed{f\left(\frac{1}{3}\right) = -\frac{13}{20}}$$

- Calculate $f(x+h)$.

$$f(\boxed{x+h}) = \frac{2\boxed{x+h}-5}{7-\boxed{x+h}} \Rightarrow f(x+h) = \frac{2(x+h)-5}{7-(x+h)}$$

$$\boxed{f(x+h) = \frac{2x+2h-5}{7-x-h}}$$

- Calculate $f\left(\dfrac{1}{x}\right)$.

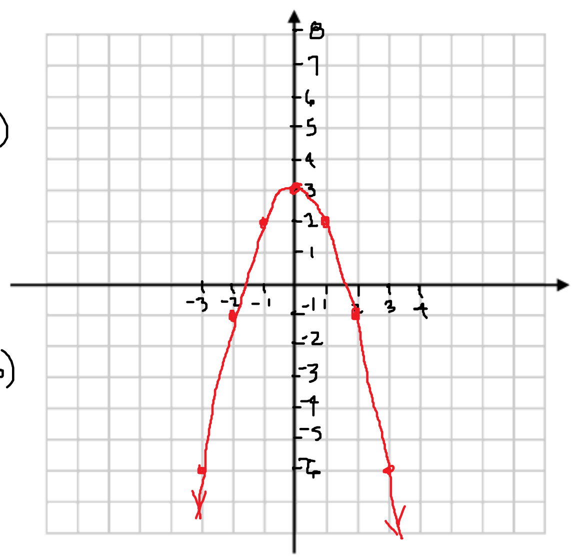
$$f\left(\frac{1}{x}\right) = \frac{2\left(\frac{1}{x}\right)-5}{7-\frac{1}{x}} \cdot \frac{x}{x} = \boxed{\frac{2-5x}{7x-1} = f\left(\frac{1}{x}\right)}$$

Definition: The graph of a function f consists of those ordered pairs (x, y) such that x is in the domain of f and $y = f(x)$.

Graphing a function by plotting points:

Example 8: Sketch the graph of $f(x) = 3 - x^2$ by plotting points. Determine its domain and range from the graph.

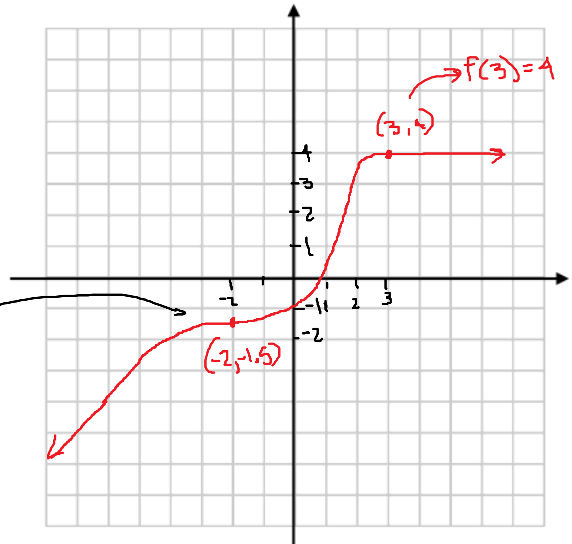
x	$f(x) = 3 - x^2$	
-3	$f(-3) = 3-(-3)^2 = 3-9 = -6$	$\Rightarrow (-3,-6)$
-2	$f(-2) = 3-(-2)^2 = 3-4 = -1$	$\Rightarrow (-2,-1)$
-1	$f(-1) = 3-(-1)^2 = 3-1 = 2$	$\Rightarrow (-1,2)$
0	$f(0) = 3-0^2 = 3$	$\Rightarrow (0,3)$
1	$f(1) = 3-(1)^2 = 3-1 = 2$	$\Rightarrow (1,2)$
2	$f(2) = 3-(2)^2 = 3-4 = -1$	$\Rightarrow (2,-1)$
3	$f(3) = 3-(3)^2 = 3-9 = -6$	$\Rightarrow (3,-6)$



Plotting points is a useful way to graph functions, but it has limitations. You don=t know for sure what the function is doing in between the points you plotted.

Understanding the graph of a function:

Example 9:



State the x- and y-intercepts.

x-intercept: ≈ 0.8 or $\approx (0.8, 0)$

y-intercept: -1 or $(0, -1)$

What is $f(-2)$? $f(-2) = -1.5$

What is $f(3)$? $f(3) = 4$

What is $f(0)$? $f(0) = -1$

(Corresponds to y-intercept $(0, -1)$)

Important: When graphing, you must have a scale on both axes

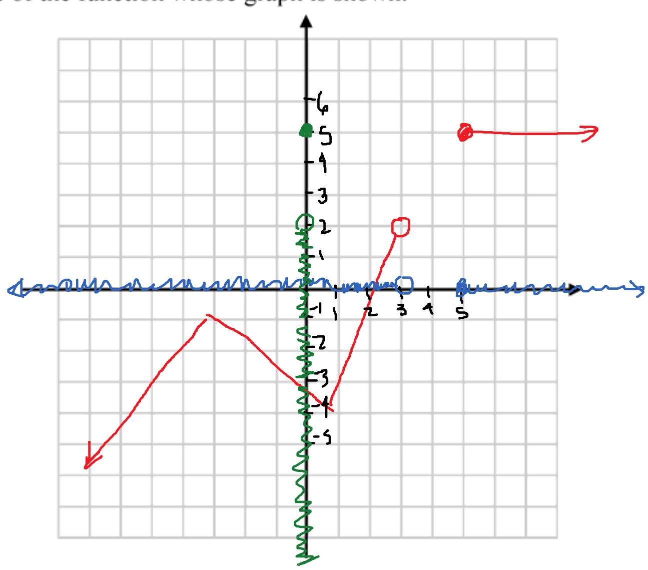
Finding domain and range from a graph:

- Domain: the set of x-values that correspond to a point on the graph. Set of inputs
- Range: the set of y-values that correspond to points on the graph. Set of outputs

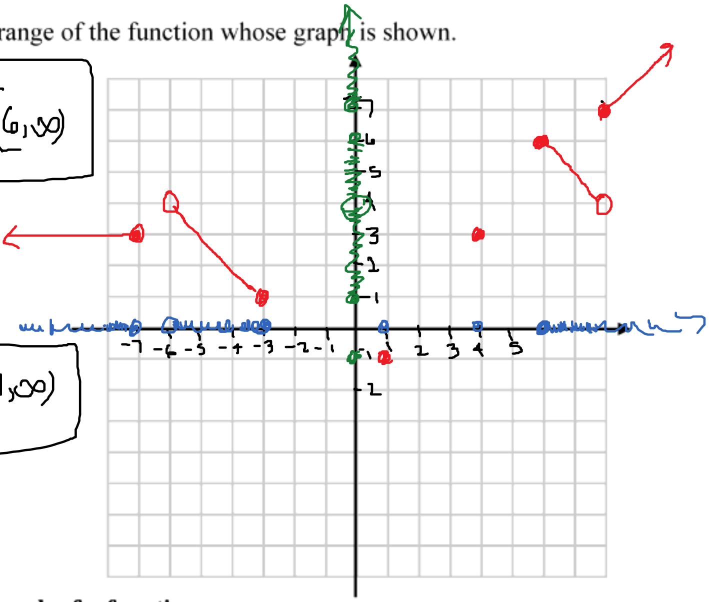
Example 10: Find the domain and range of the function whose graph is shown.

Domain: $(-\infty, 3) \cup [5, \infty)$

Range: $(-\infty, 2) \cup \{5\}$



Example 11: Find the domain and range of the function whose graph is shown.



Domain: $\boxed{(-\infty,-7] \cup (-6,3] \cup \{1,4\} \cup [6,\infty)}$

Range: $\boxed{\{-1\} \cup [1,4) \cup (4,6] \cup [7,\infty)}$

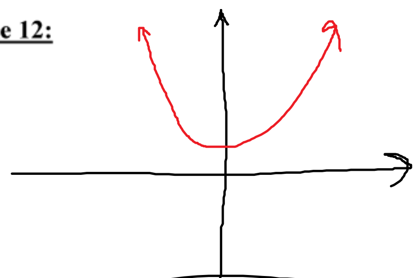
Recognizing when a graph is the graph of a function:

> Vertical Line Test:
>
> If any vertical line intersects a graph in more than one point, the graph does not define *y* as a function of *x*.

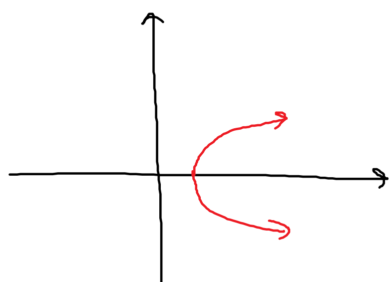
Example 12:

Are these graphs of functions?

Does this graph define y as a function of x



Yes, this is the graph of a function



No, this is not the graph of a function