2.2: More on Functions and Their Graphs

The difference quotient:

The expression $\frac{f(x+h)-f(x)}{h}$ is called a *difference quotient* for *f*. The difference quotient is important in our understanding of the rate of change of a function.

Example 1: For the function $f(x)=7-3x$, find and simplify the difference quotient.

Example 2: For the function $f(x)=x^2+2x-1$, find and simplify the difference quotient.

Piecewise Defined Functions:
(Functions defined in "pieces")

<u>Example 3:</u> $f(x)=\begin{cases} 3-x, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$

There are <u>two different rules</u>. The rule we use depends on which *x* we put in.

Calculate $f(4),\ f(-5),\ f(1)$.

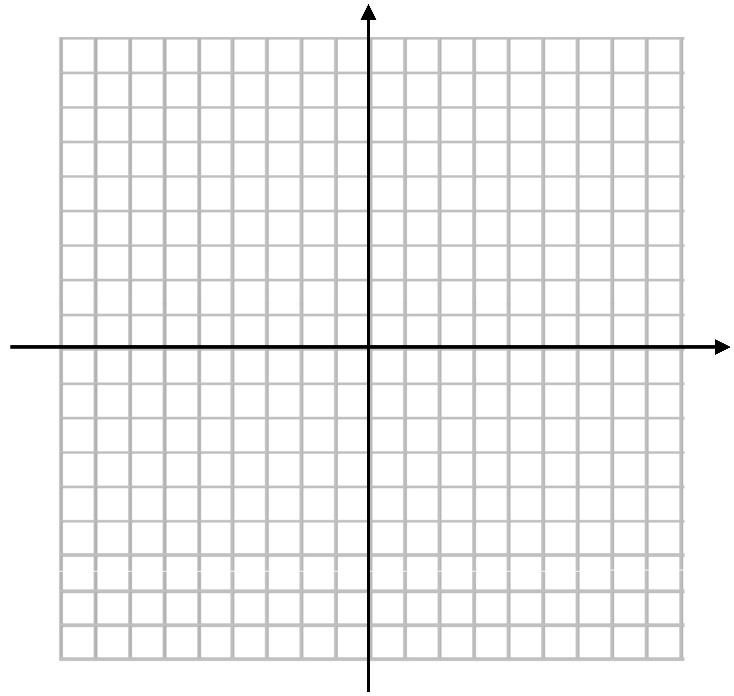
<u>Example 4:</u> Write $f(x)=|x|$ as a piecewise function.

<u>Example 5:</u> $f(x)=\begin{cases} 3x+4, & \text{if } x < -2 \\ 7 & \text{if } x = -2 \\ x^2+1, & \text{if } x > -2 \end{cases}$

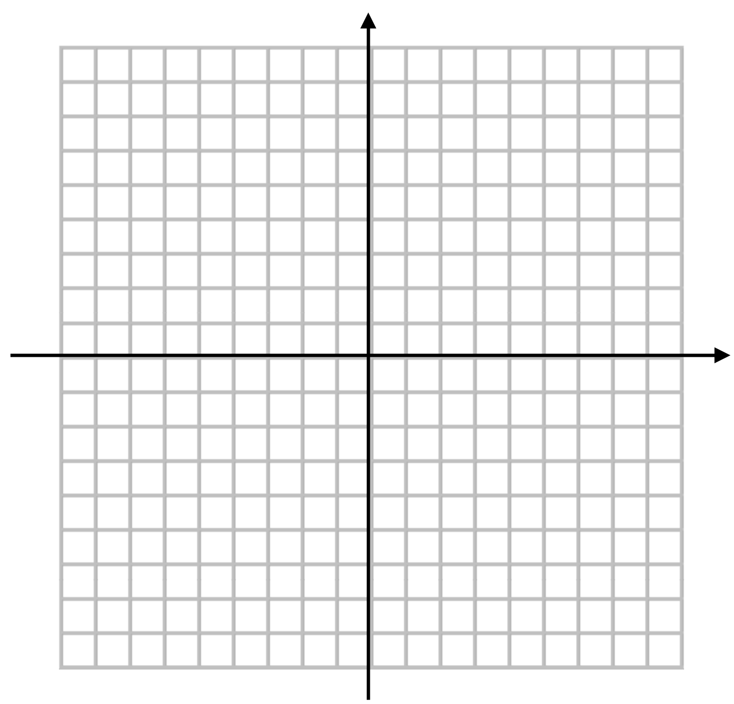
Calculate $f(3),\ f(-3),\ f(-2)$.

Graphing piecewise-defined functions: (Piece functions)

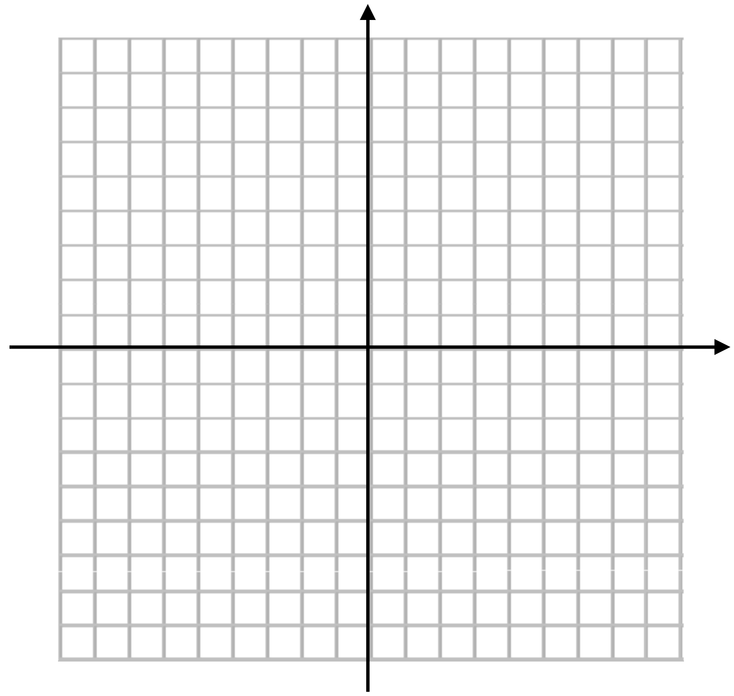
Example 1: $f(x)=\begin{cases} 3 & \text{if } x \le 1 \\ -2 & \text{if } x > 1 \end{cases}$



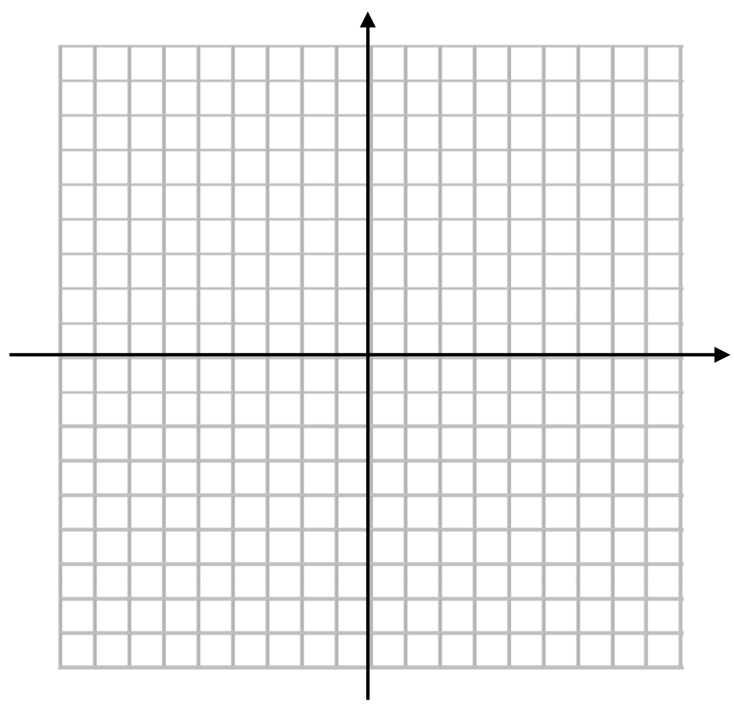
Example 2: $f(x)=\begin{cases} x & \text{if } x \ge 0 \\ -x+2 & \text{if } x < 0 \end{cases}$



Example 3: $g(x)=\begin{cases} -3 & \text{if } x>2 \\ x^2 & \text{if } x<2 \\ 1 & \text{if } x=2 \end{cases}$



Example 4: $f(x)=\begin{cases} x^3 & \text{if } x\geq 0 \\ 3x+4 & \text{if } x<0 \end{cases}$



Symmetry:

A graph is *symmetric with respect to the x-axis* if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is *symmetric with respect to the y-axis* if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

A graph is *symmetric with respect to the origin* if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

Symmetry about the *x*-axis:

Symmetry about the *y*-axis:

Symmetry about the origin:

Example 6: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$y = x^2 + 8$$

Example 7: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$x = y^2 + 8$$

Example 8: Is this equation symmetric around (a) the *x*-axis? (b) the *y*-axis? (c) the origin?

$$x^2 = y^2 + 8$$

Even and odd functions:

A function f is an <u>even function</u> if $f(-x) = f(x)$ for every *x* in the domain of *f*. The graph of an even function is symmetric with respect to the *y*-axis.

A function f is an <u>odd function</u> if $f(-x) = -f(x)$ for every *x* in the domain of *f*. The graph of an even function is symmetric with respect to the origin.

<u>Example 9:</u> Is the function $f(x) = 2 + x^2 + 3x^4$ even, odd, or neither?

<u>Example 10:</u> Is the function $f(x) = x^3 + x^5$ even, odd, or neither?

<u>Example 11:</u> Is the function $f(x) = x^3 + 1$ even, odd, or neither?

<u>Example 12:</u> State whether each is the graph of a function that is odd, even or neither.