

2.5: Transformations of Functions

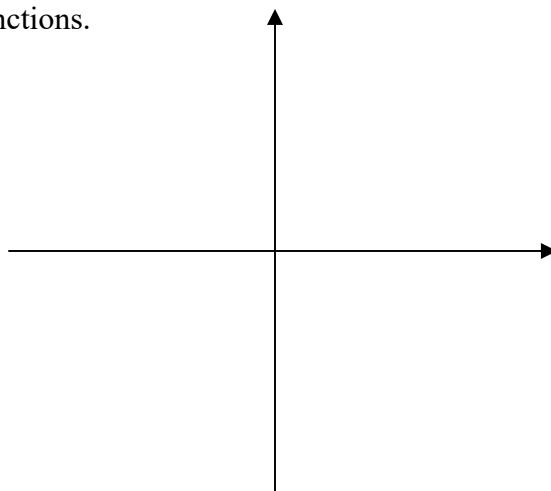
Learn the graphs of these frequently encountered functions.

Constant function $f(x) = c$

Domain:

Range:

Is it odd or even?

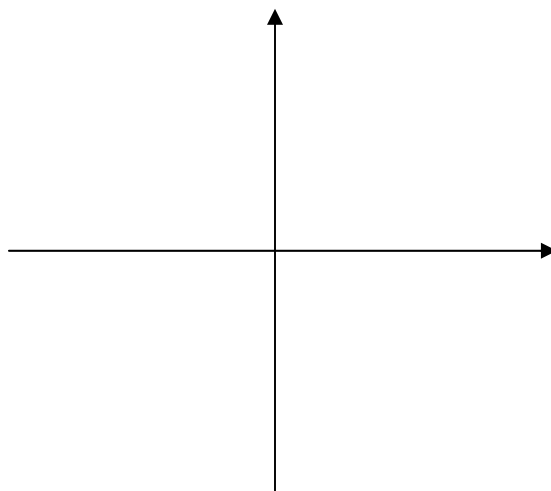


Identity function $f(x) = x$

Domain:

Range:

Is it odd or even?

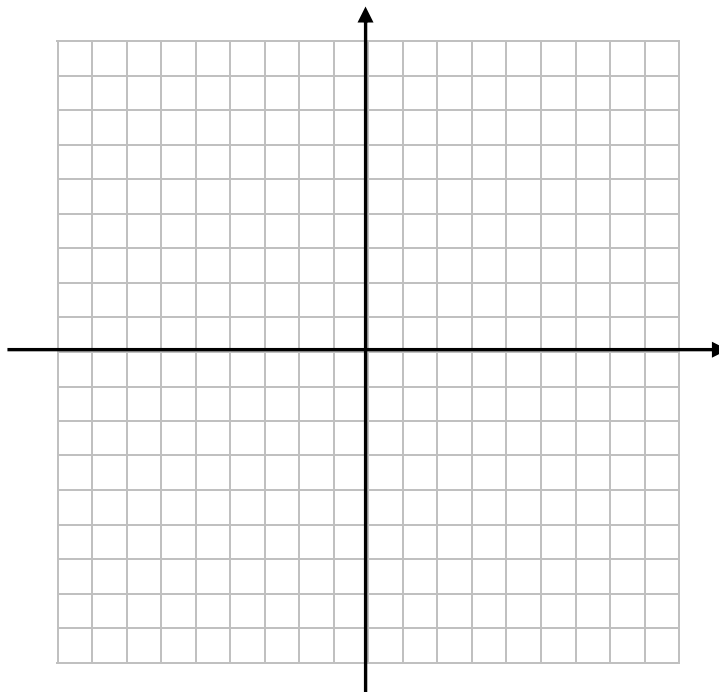


Standard quadratic function $f(x) = x^2$

Domain:

Range:

Is it odd or even?

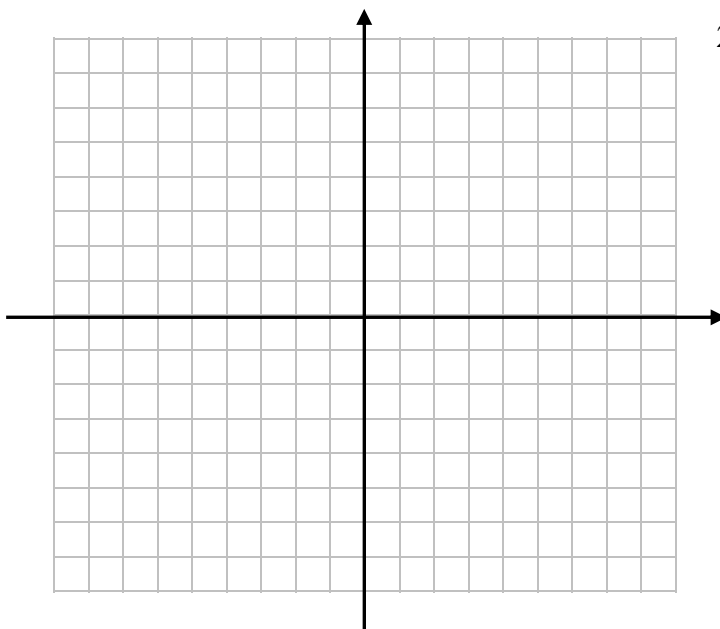


Standard cubic function $f(x) = x^3$

Domain:

Range:

Is it odd or even?

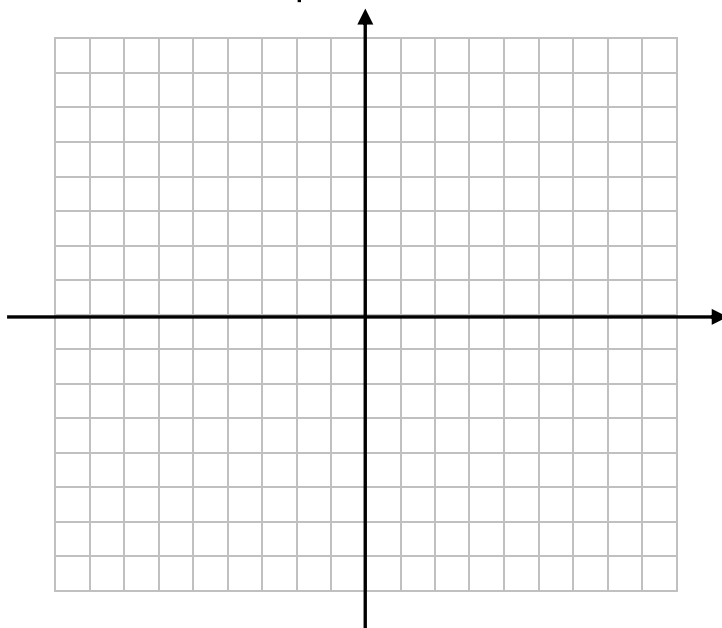


Square root function $f(x) = \sqrt{x}$:

Domain:

Range:

Is it odd or even?

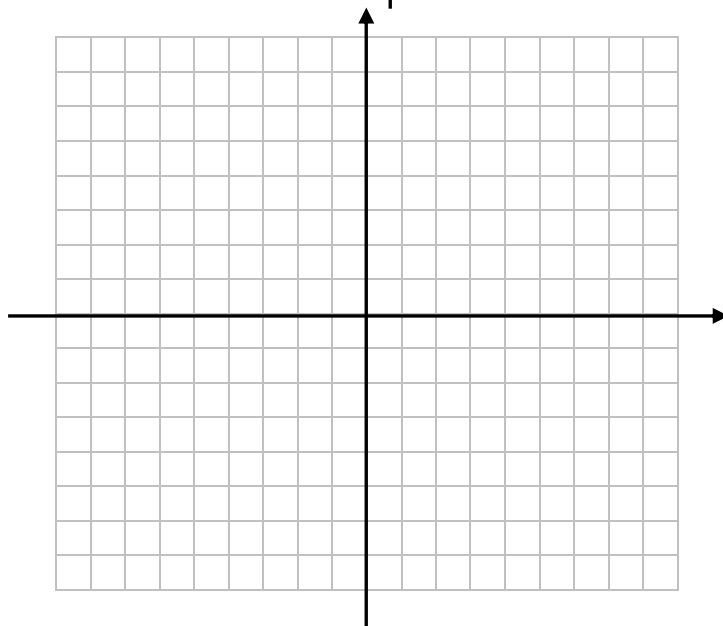


Absolute value function $f(x) = |x|$

Domain:

Range:

Is it odd or even?



If the graph of a function is known, similar functions can be graphed by varying them in several ways.

- Vertical translation (shifting vertically)
- Horizontal translation (shifting horizontally)
- Reflecting about the x -axis
- Reflecting about the y -axis
- Vertical stretching and shrinking

Translation of functions:

To *translate* a graph means to shift it horizontally, vertically, or both.

Horizontal Translation:

- Replacing x by $x - c$, with c positive, shifts the graph c units to the right.
- Replacing x by $x + c$, with c positive, shifts the graph c units to the left.

Vertical Translation:

- Adding a positive number d to the function shifts the graph upward by d units.
- Subtracting a positive number d from the function shifts the graph downward by d units.

Note:

- Adding a positive number d to the function (upward shift) is equivalent to replacing y by $y - d$.

$$y = f(x) + d$$

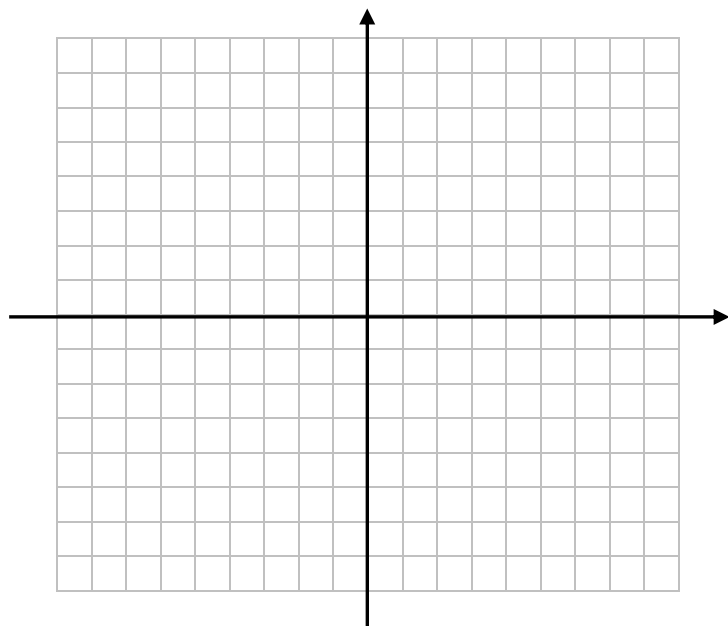
$$y - d = f(x)$$

- Subtracting a positive number d (downward shift) from the function is equivalent to replacing y by $y + d$.

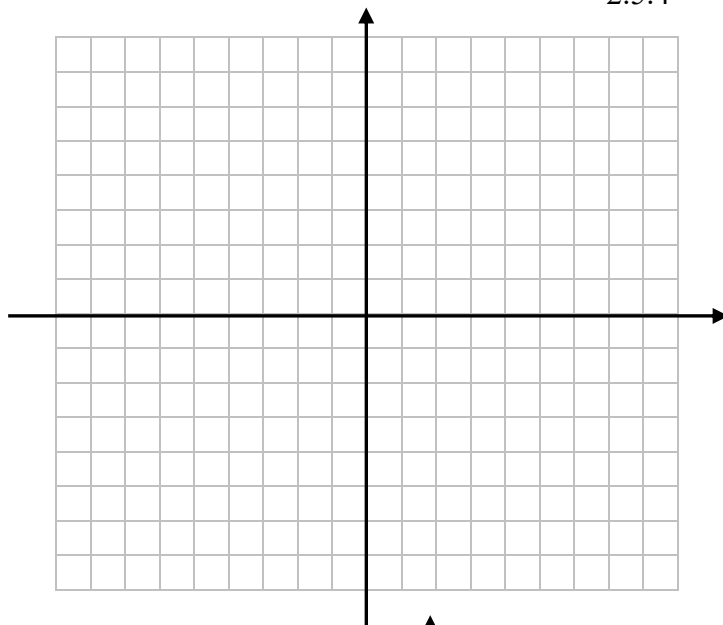
$$y = f(x) - d$$

$$y + d = f(x)$$

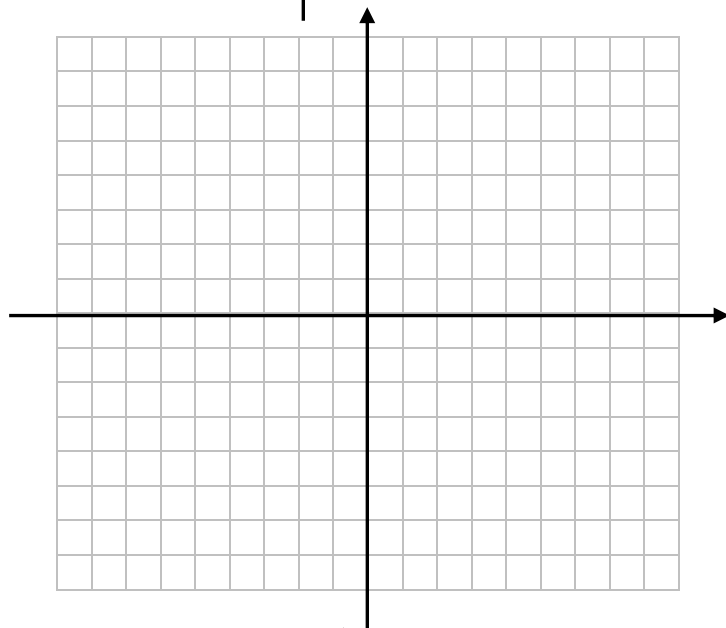
Example 1: Sketch the graph of $f(x) = x^2 - 3$.



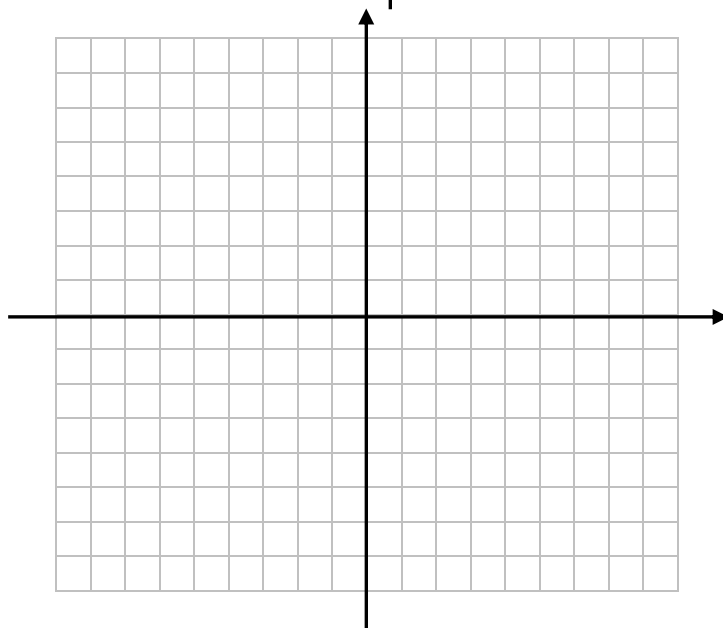
Example 2: Sketch the graph of $g(x) = (x + 2)^2$.



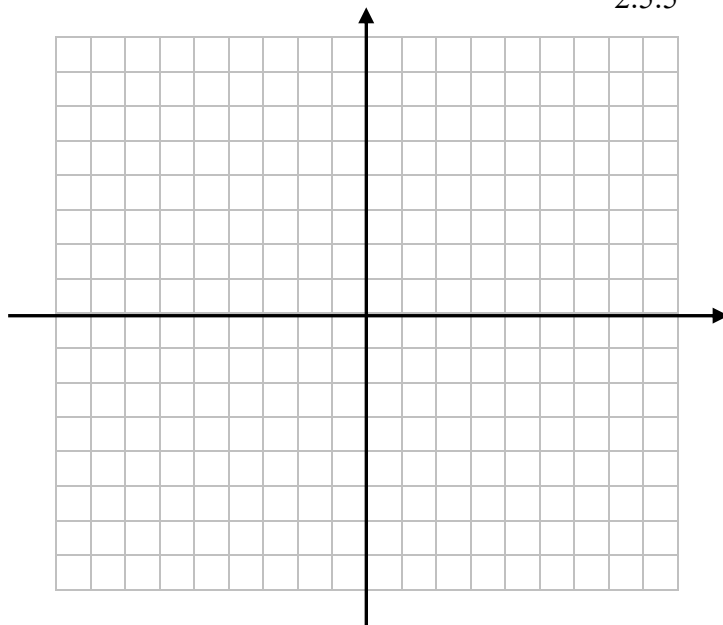
Example 3: Sketch the graph of $f(x) = |x - 4|$.



Example 4: Sketch the graph of $f(x) = (x + 2)^2 + 1$.



Example 5: Sketch the graph of $y = \sqrt{x-2} - 3$.

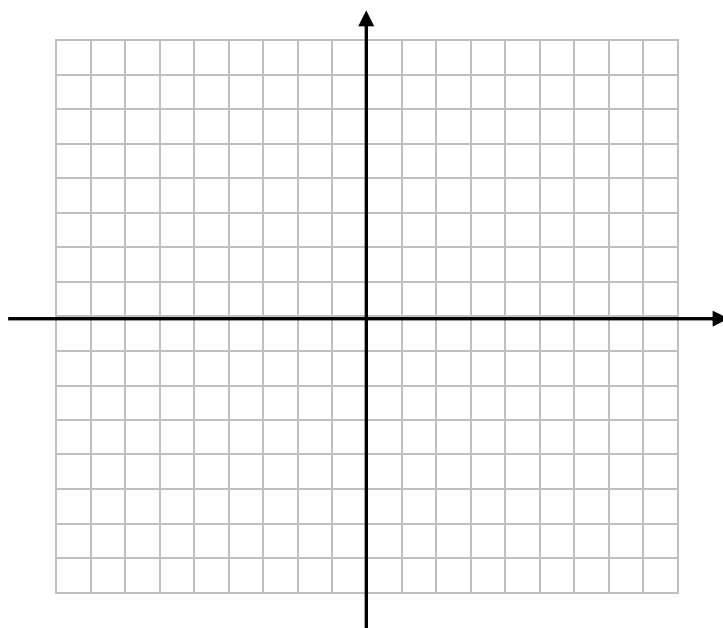


Reflection of functions:

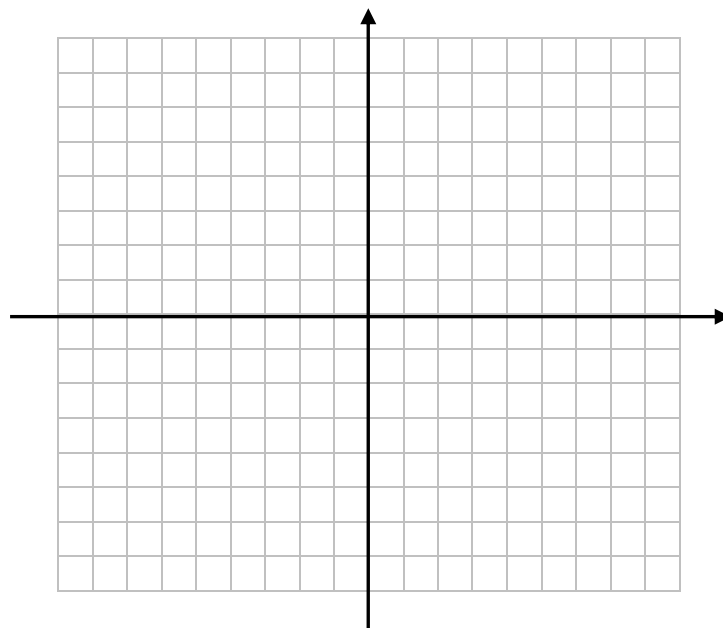
A *reflection* is the “mirror-image” of graph about the x -axis or y -axis.

- Changing $f(x)$ to $-f(x)$ (multiplying $f(x)$ by -1 reflects the graph about the x -axis.
(e.g. changing $y = \sqrt{x}$ to $y = -\sqrt{x}$)
- Changing $f(x)$ to $f(-x)$ (replacing x by $-x$) reflects the graph about the y -axis.
(e.g. changing $y = \sqrt{x}$ to $y = \sqrt{-x}$)

Example 6: Sketch the graph of $f(x) = -\sqrt{x}$.



Example 7: Sketch the graph of $f(x) = \sqrt{-x}$.

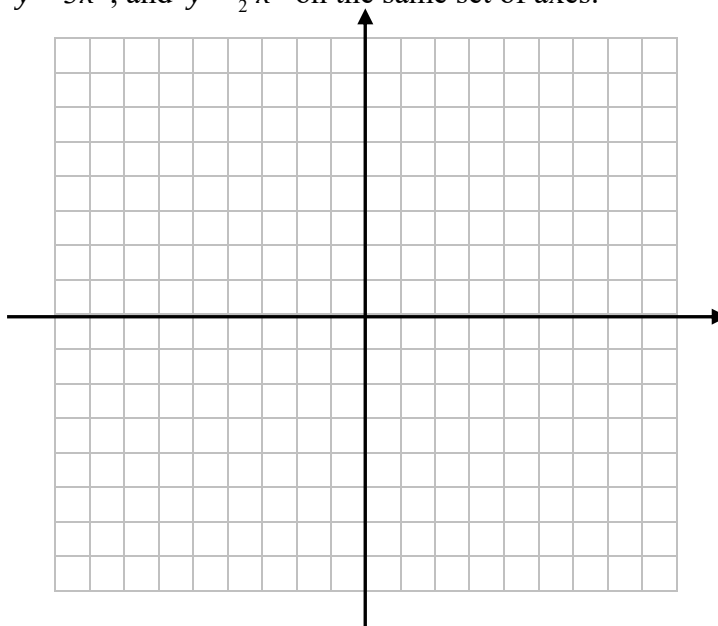


Vertical stretching and shrinking:

Multiplying a function by a number a will multiply all the y -values by a .

- Multiplying a function by a , with $a > 1$, “stretches” the graph vertically.
- Multiplying a function by a , with $0 < a < 1$, “shrinks” the graph vertically.

Example 8: Sketch the graphs of $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$ on the same set of axes.



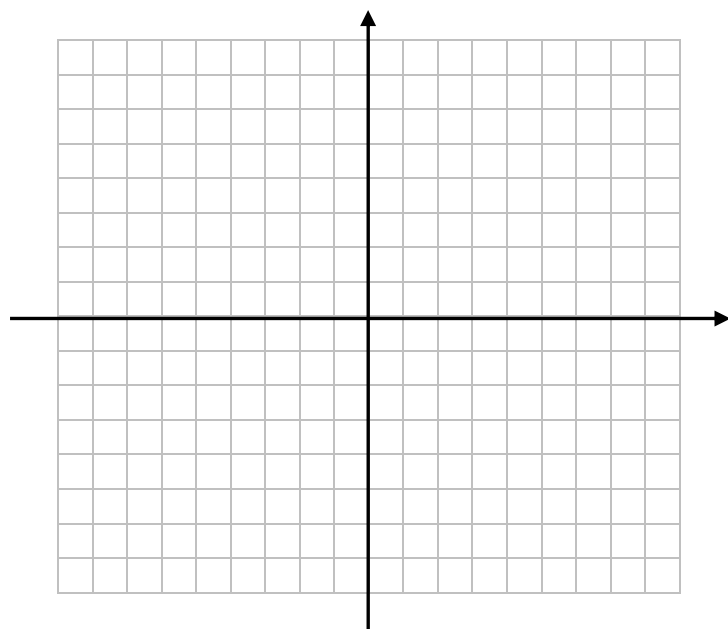
Combinations of transformations:

Recommended order for transformations:

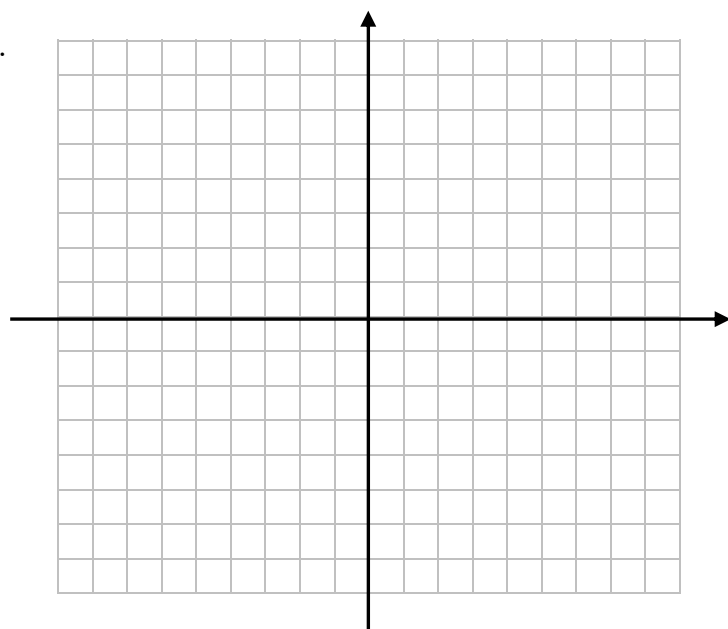
1. Stretching and shrinking
2. Reflection about x -axis.
3. Translations (horizontal and vertical).
4. Reflection about y -axis. (important to do this last!)

This is not the only order that works, but it is safest, and avoids having to perform algebraic manipulation of negative signs, etc.

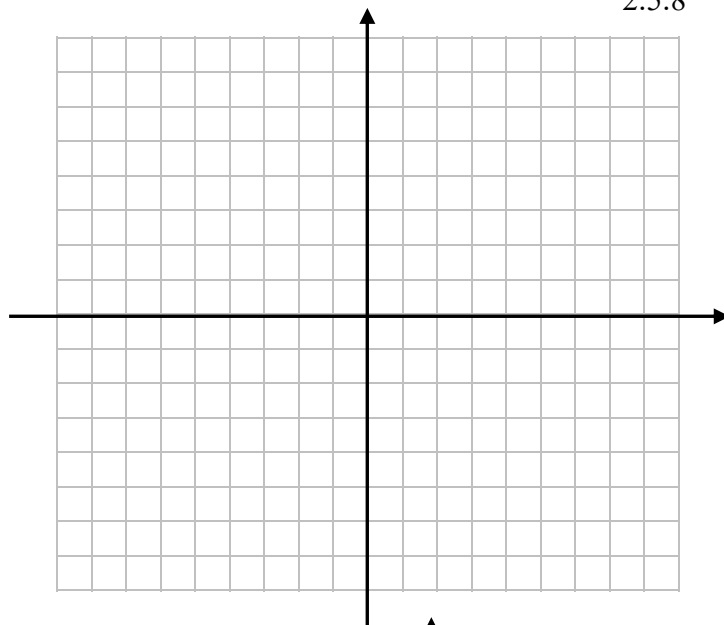
Example 9: Sketch the graph of $f(x) = 4|x - 3| + 1$.



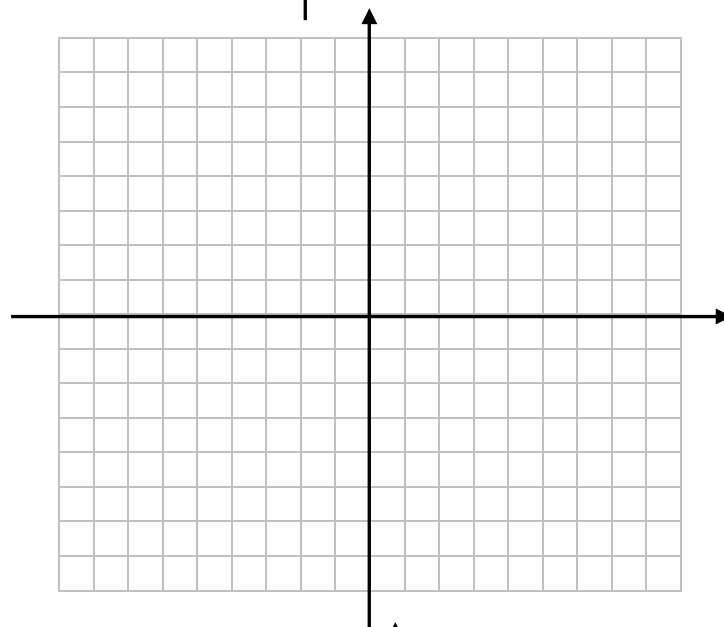
Example 10: Sketch the graph of $f(x) = -(x + 2)^2 + 4$.



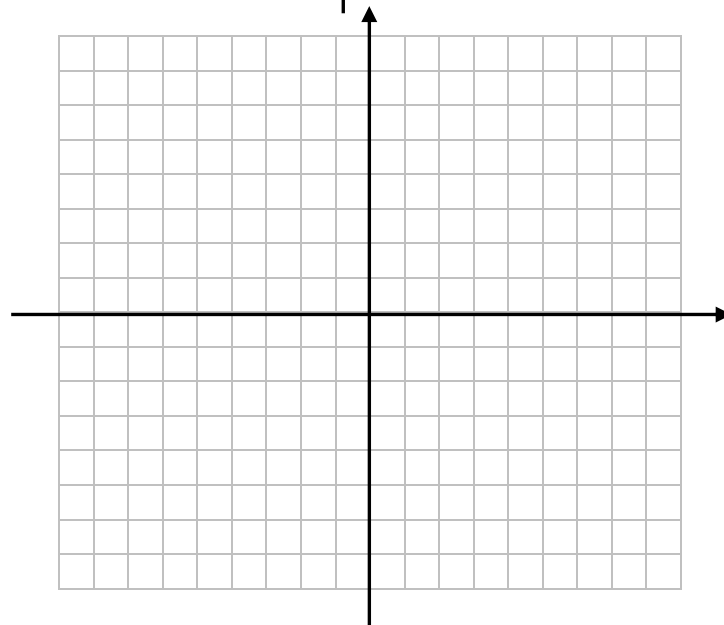
Example 11: Sketch the graph of $y = -\sqrt{x} - 3$.



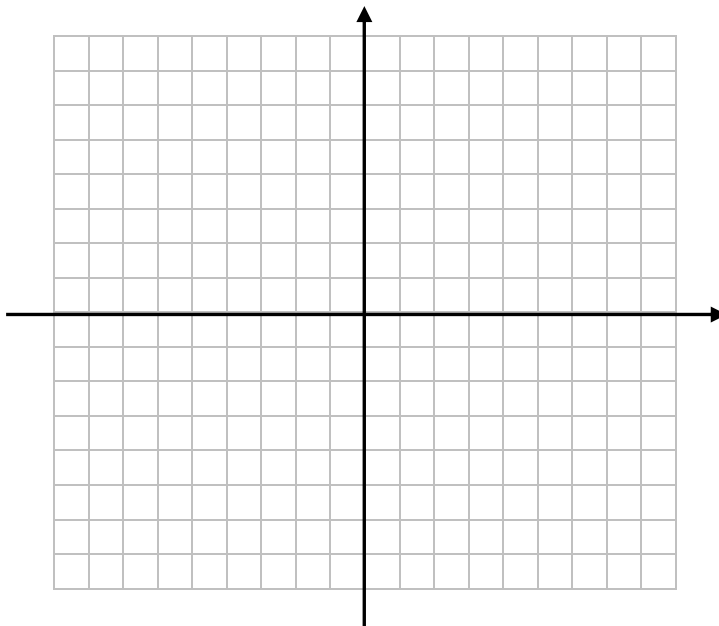
Example 12: Sketch the graph of $y = \frac{1}{2}x^3 - 2$.



Example 13: Sketch the graph of $g(x) = -(x+1)^3 - 2$.



Example 14: Sketch the graph of $y = \sqrt{5-x} + 2$.



Example 15: Given the graph of $f(x)$, sketch the graph of

a. $-f(x)$

b. $f(-x)$

c. $f(x-2)$

