1314-2-5-Notes-transformations

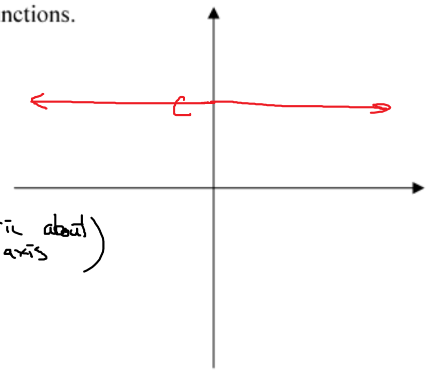
Thursday, September 26, 2019 10:52 AM

1314-2-5-Notes-transformations

2.5: Transformations of Functions

Learn the graphs of these frequently encountered functions.

Constant function $f(x)=c$



Domain: $(-\infty, \infty)$

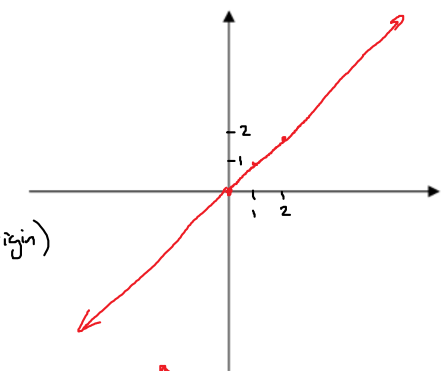
Range: $\{c\}$

Is it odd or even? even function (symmetric about y-axis)

Ex. $f(x) = -2$

x	$f(x)$
-3	$f(-3) = -2 \Rightarrow (-3,-2)$
-2	$f(-2) = -2 \Rightarrow (-2,-2)$
-1	$f(-1) = -2$
0	$f(0) = -2$
1	$f(1) = -2$
2	-2
3	-2

Identity function $f(x)=x$



Domain: $(-\infty, \infty)$

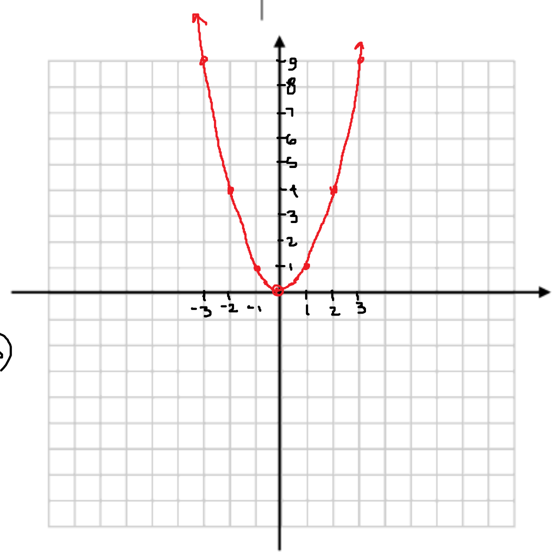
Range: $(-\infty, \infty)$

Is it odd or even? odd function (symmetric about origin)

ordered pairs:
$(0,0), (1,1), (2,2), (3,3)$
$(-2,-2), (-5,-5)$

OR, using $y = mx + b$
$y = x$
$y = 1x + 0 \Rightarrow$ slope: $m = 1 = +\frac{1}{1}$
y-int: $b = 0 \Rightarrow (0,0)$

Standard quadratic function $f(x)=x^2$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even? even function (symmetric around y-axis)

x	$f(x) = x^2$
-3	$(-3)^2 = 9$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$
3	$(3)^2 = 9$

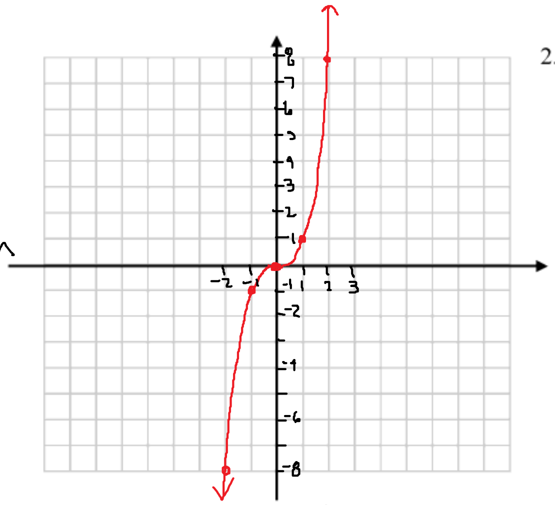
Standard cubic function $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Is it odd or even? odd function (symmetric about the origin)

x	$f(x) = x^3$
-3	$(-3)^3 = -27$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$(0)^3 = 0$
1	$(1)^3 = 1$
2	$(2)^3 = 8$
3	$(3)^3 = 27$



Square root function $f(x) = \sqrt{x}$:

Domain: $[0, \infty)$

Range: $[0, \infty)$

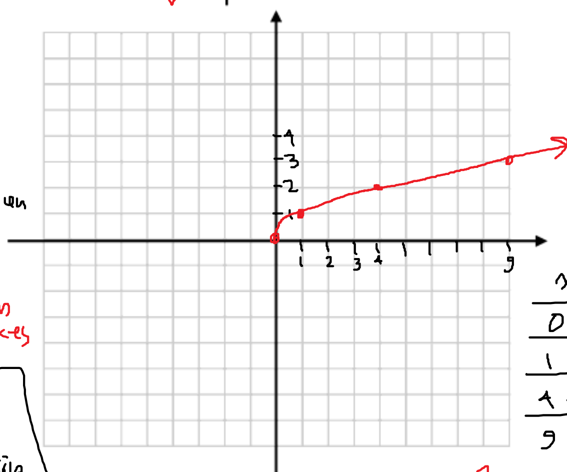
Is it odd or even? neither odd nor even

x	$f(x) = \sqrt{x}$
-3	$\sqrt{-3} = i\sqrt{3}$ not a real number
-2	not real
-1	not real
0	$f(0) = \sqrt{0} = 0$
1	$f(1) = \sqrt{1} = 1$
4	$f(4) = \sqrt{4} = 2$
9	$f(9) = \sqrt{9} = 3$

not valid inputs

cannot be used for graphing on x and y axes

In sections on graphing functions, we will only consider real-valued functions



Know these

x	$f(x)$
0	0
1	1
4	2
9	3

Absolute value function $f(x) = |x|$

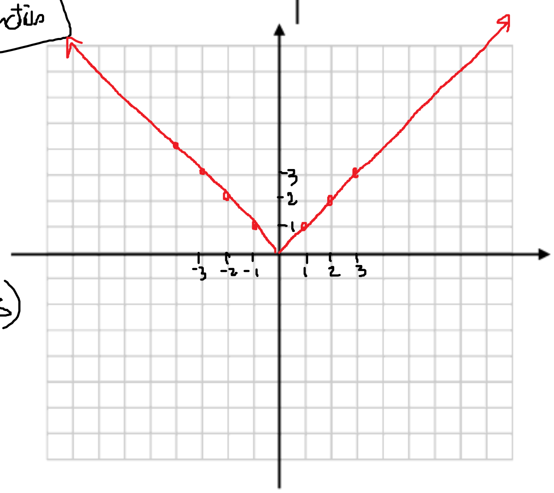
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even?

even function (symmetric around y-axis)

x	$f(x) = \lvert x \rvert$
-3	$\lvert -3 \rvert = 3$
-2	$\lvert -2 \rvert = 2$
-1	$\lvert -1 \rvert = 1$
0	$\lvert 0 \rvert = 0$
1	$\lvert 1 \rvert = 1$
2	$\lvert 2 \rvert = 2$
3	$\lvert 3 \rvert = 3$



If the graph of a function is known, similar functions can be graphed by varying them in several ways.

- Vertical translation (shifting vertically)
- Horizontal translation (shifting horizontally)
- Reflecting about the x-axis
- Reflecting about the y-axis
- Vertical stretching and shrinking

Translation of functions:

To *translate* a graph means to shift it horizontally, vertically, or both.

Horizontal Translation:

- Replacing x by $x-c$, with c positive, shifts the graph c units to the right.
- Replacing x by $x+c$, with c positive, shifts the graph c units to the left.

Note: $x+c=0$, $x=-c$ | $x-c=0$, $x=c$

Vertical Translation:

- Adding a positive number d to the function shifts the graph upward by d units.
- Subtracting a positive number d from the function shifts the graph downward by d units.

Note:

- Adding a positive number d to the function (upward shift) is equivalent to replacing y by $y-d$.

$$y = f(x)+d$$
$$y-d = f(x)$$

- Subtracting a positive number d (downward shift) from the function is equivalent to replacing y by $y+d$.

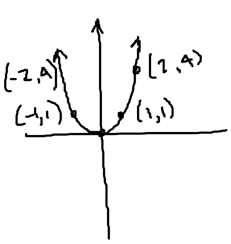
$$y = f(x)-d$$
$$y+d = f(x)$$

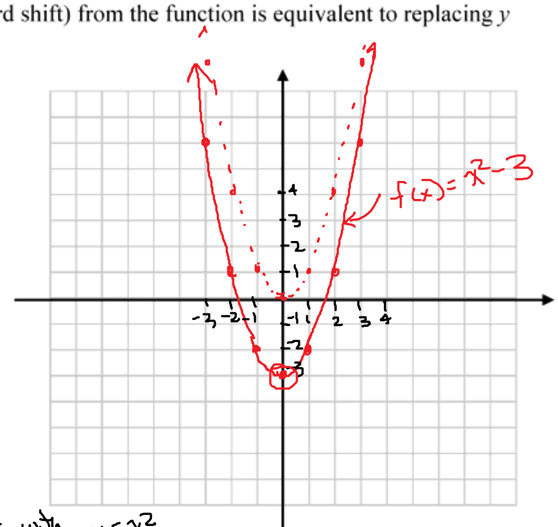
Example 1: Sketch the graph of $f(x) = x^2 - 3$.

Determine the "parent function" this is created from

$y = x^2$

x	$y=x^2$
3	9
2	4
1	1
0	0
-1	1
-2	4
-3	9



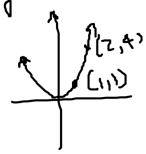


Start with $y=x^2$
Shift (translate) it Down 3

Example 2: Sketch the graph of $g(x) = (x+2)^2$.

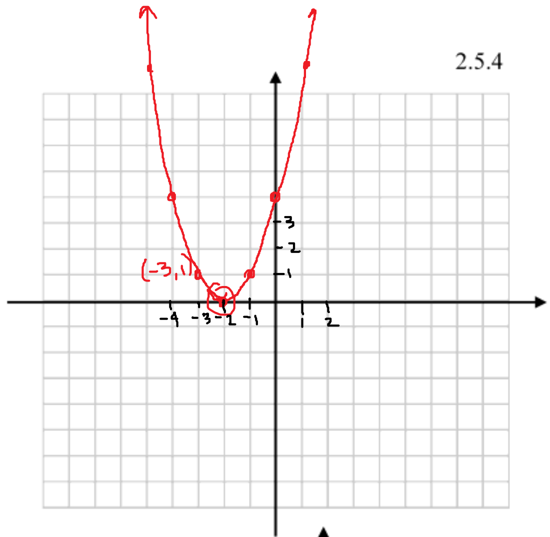
Parent function:

$y = x^2$, (3, 9)



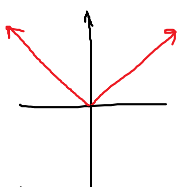
Start with $y = x^2$.
Then shift it left 2.

Why move left?
$x + 2 = 0$
$x = -2$

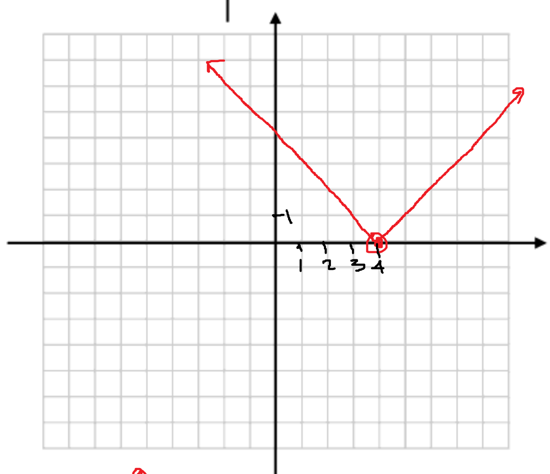


Example 3: Sketch the graph of $f(x) = |x-4|$.

Parent function: $y = |x|$

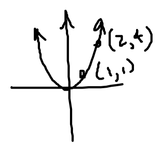


Start with $y = |x|$ then shift it right 4
(because $x - 4 = 0$
$x = 4$)



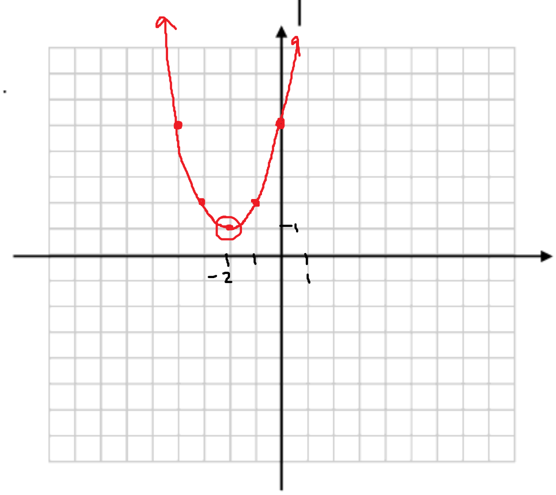
Example 4: Sketch the graph of $f(x) = (x+2)^2 + 1$.

Parent function: $y = x^2$
Shift it left 2, up 1



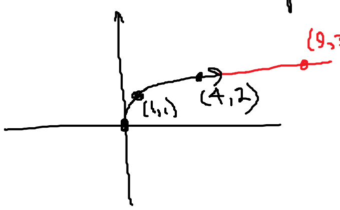
$y = (x+2)^2 + 1$
$y - 1 = (x+2)^2$

$x + 2 = 0$
$x = -2$
$y - 1 = 0$
$y = 1$



Example 5: Sketch the graph of $y=\sqrt{x-2}-3$.

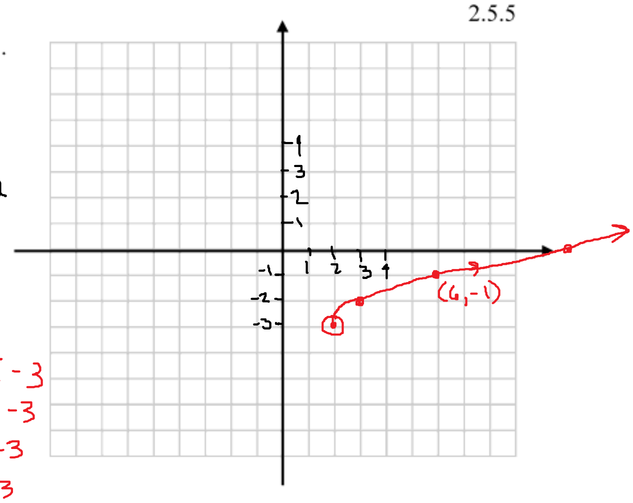
Parent function: $y=\sqrt{x}$



shift it: right 2

Note: $x-2=0$
$x=2$

down 3



Check. Is $(6,-1)$ on the graph of $y=\sqrt{x-2}-3$

$x=6 \Rightarrow y=\sqrt{6-2}-3$

$y=\sqrt{4}-3$

$y=2-3$

$y=-1 \Rightarrow$ yes $(6,-1)$ on graph

Reflection of functions:

A *reflection* is the "mirror-image" of graph about the x-axis or y-axis.

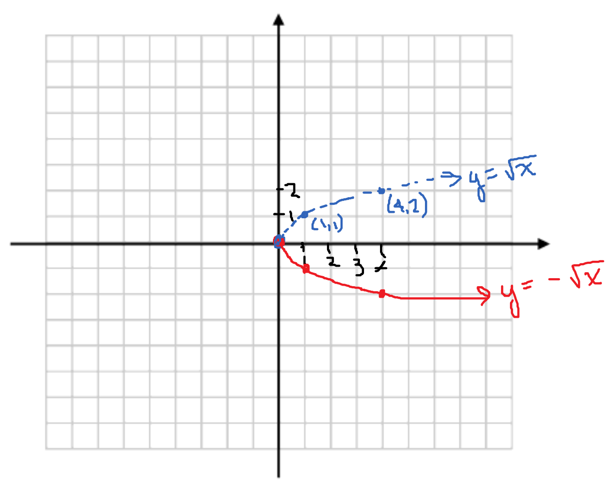
- Changing $f(x)$ to $-f(x)$ (multiplying $f(x)$ by -1 reflects the graph about the x-axis.
  (e.g. changing $y=\sqrt{x}$ to $y=-\sqrt{x}$)

- Changing $f(x)$ to $f(-x)$ (replacing x by $-x$) reflects the graph about the y-axis.
  (e.g. changing $y=\sqrt{x}$ to $y=\sqrt{-x}$)

Example 6: Sketch the graph of $f(x)=-\sqrt{x}$.

Parent Function: $y=\sqrt{x}$

Reflect it across x-axis



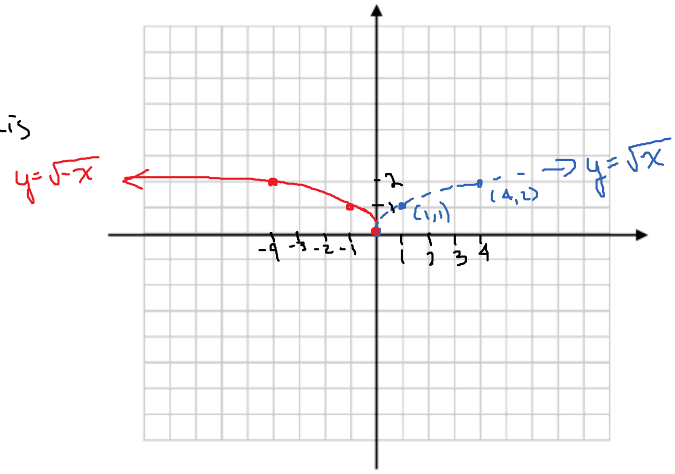
Example 7: Sketch the graph of $f(x) = \sqrt{-x}$.

Parent function: $y = \sqrt{x}$

Reflect it around the y-axis

Domain of $f(x) = \sqrt{-x}$: $(-\infty, 0]$

Range of $f(x) = \sqrt{-x}$: $[0, \infty)$



Vertical stretching and shrinking:

Multiplying a function by a number a will multiply all the y-values by a.

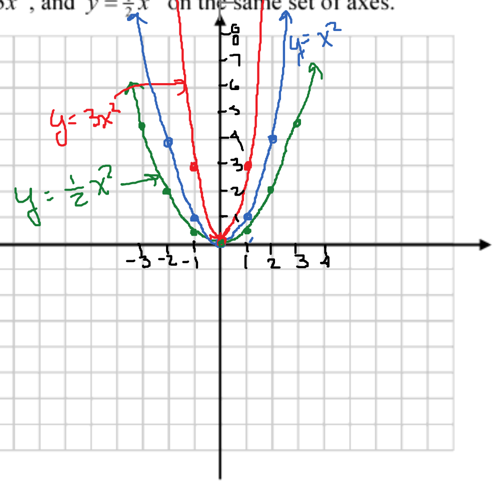
- Multiplying a function by a, with $a > 1$, "stretches" the graph vertically.
- Multiplying a function by a, with $0 < a < 1$, "shrinks" the graph vertically.

Example 8: Sketch the graphs of $y = x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$ on the same set of axes.

All have parent function $y = x^2$

For $y = 3x^2$: multiply the y-values by 3

For $y = \frac{1}{2}x^2$, start with $y = x^2$ and multiply the y-values by $\frac{1}{2}$

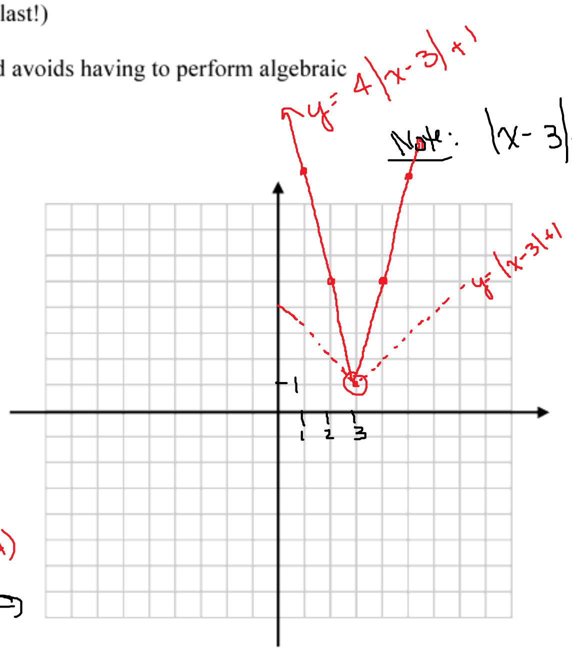


Combinations of transformations:

Recommended order for transformations:

1. Stretching and shrinking
2. Reflection about x-axis.
3. Translations (horizontal and vertical).
4. Reflection about y-axis. (important to do this last!)

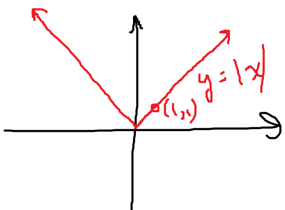
This is not the only order that works, but it is safest, and avoids having to perform algebraic manipulation of negative signs, etc.



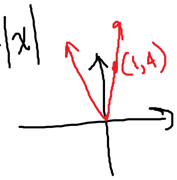
Note: $|x-3| \neq x+3$

Example 9: Sketch the graph of $f(x) = 4|x-3|+1$.

Parent function: $y = |x|$

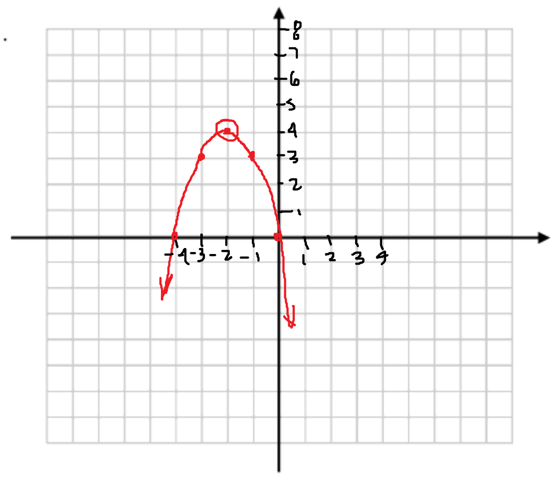


Multiply y-values by 4: $y = 4|x|$

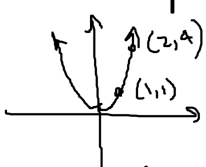


Shift this right 3, up 1

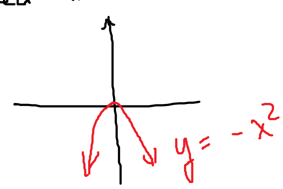
Example 10: Sketch the graph of $f(x) = -(x+2)^2 + 4$.



Parent function: $y = x^2$



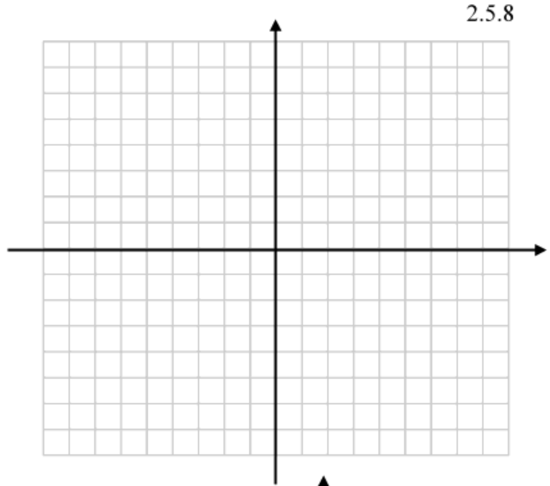
Then reflect it around x-axis:



Then shift it left 2, up 4

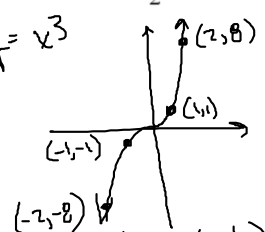
Example 11: Sketch the graph of $y = -\sqrt{x} - 3$.

Basic function



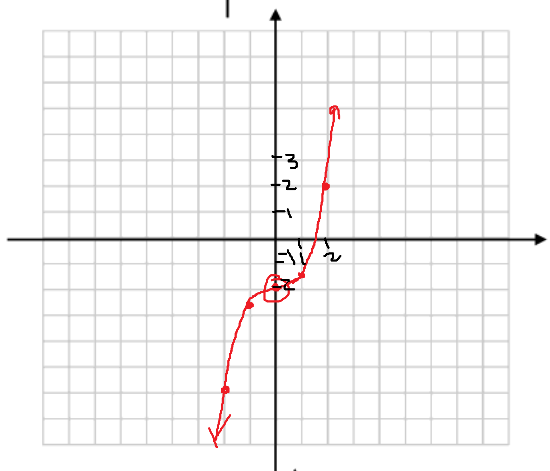
Example 12: Sketch the graph of $y = \frac{1}{2}x^3 - 2$.

Parent function: $y = x^3$

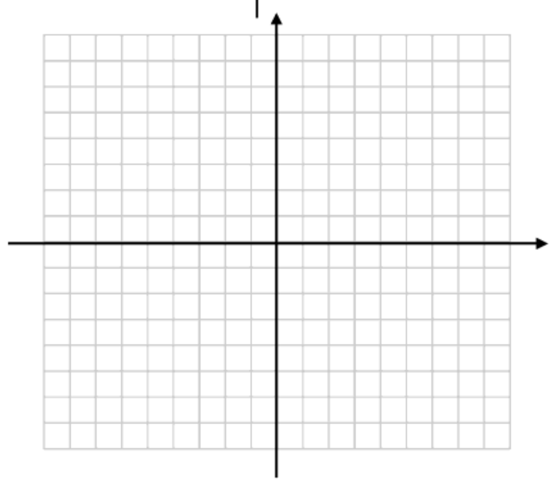


Then multiply our y-values by $\frac{1}{2} \Rightarrow (1, \frac{1}{2})$
$(2, 4)$

Then shift down 2

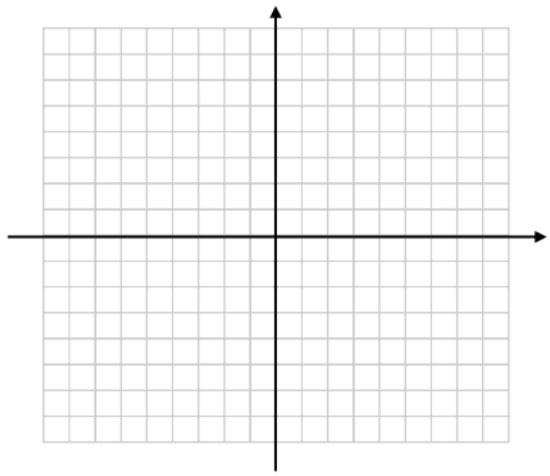


Example 13: Sketch the graph of $g(x) = -(x+1)^3 - 2$.



2.5.9

Example 14: Sketch the graph of $y = \sqrt{5-x} + 2$.

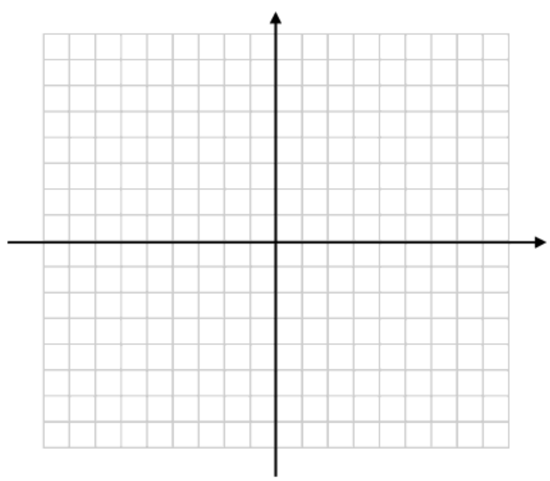


Example 15: Given the graph of $f(x)$, sketch the graph of

a. $-f(x)$

b. $f(-x)$

c $f(x-2)$



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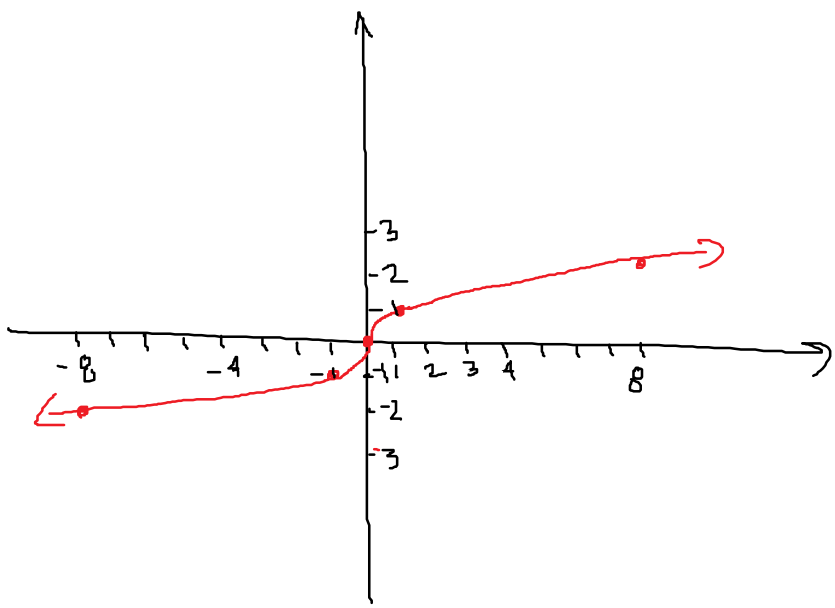

$f(x) = \sqrt[3]{x}$

x	$f(x) = \sqrt[3]{x}$
-27	$\sqrt[3]{-27} = -3$
-8	$\sqrt[3]{-8} = -2$
-1	$\sqrt[3]{-1} = -1$
0	$\sqrt[3]{0} = 0$
1	$\sqrt[3]{1} = 1$
8	$\sqrt[3]{8} = 2$
27	$\sqrt[3]{27} = 3$