2.6: Combinations of Functions; Composite Functions

Domain of a function:

The *domain* is the set of all valid inputs for the function.

Sometimes the domain is stated specifically.

Example 1: $f(x) = 3x + 5; \qquad -3 \leq x < 2$

Here the domain is the interval [-3, 2).

Usually the domain of a function will *not* be specifically stated. We have to figure it out.

By convention, the domain is the set of all real numbers for which the expression is defined as a real number.

This means we=ll eliminate anything that results in zero denominators or even roots of negative numbers. Later, we=ll learn some more things that need to be eliminated.

Example 2: State the domain of $f(x) = 3x - 4$.

Example 3: State the domain of $h(x) = -17$.

Example 4: State the domain of $f(x) = \dfrac{5}{x-3}$.

Example 5: State the domain of $g(x) = \dfrac{1}{x^2 - x}$.

Example 6: State the domain of $g(x) = \dfrac{1}{x^2 + 1}$.

Example 7: State the domain of $f(x)=\sqrt{5-2x}+8$.

Example 8: State the domain of $k(x)=\dfrac{x-9}{x^2-x-20}$.

Example 9: State the domain of $g(x)=\dfrac{7-2x}{\sqrt{5x-4}}$.

Example 10: State the domain of $f(x)=\sqrt{x+2}+\sqrt{x+5}$.

Example 11: State the domain of $f(x)=\dfrac{\sqrt{4x+8}}{x-5}$.

Example 12: State the domain of $h(x)=\sqrt[5]{12x-7}$.

Example 13: State the domain of $f(x)=\dfrac{\sqrt{6-2x}}{x^2-4}$.

<u>Methods for combining functions</u>:

1. Sum $\quad (f+g)(x)=f(x)+g(x)$
2. Difference $\quad (f-g)(x)=f(x)-g(x)$
3. Product $\quad (fg)(x)=f(x)g(x)$
4. Quotient $\quad \left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$ provided $g(x)\neq 0$.
5. Composition $\quad (f\circ g)(x)=f(g(x))$

Example 1: Suppose $f(x)=2x-3$, $g(x)=x^2-4x+5$, and $h(x)=2x^3$.

a) $(f+g)(x)$

b) $(f+g)(-3)$

c) $(3f-g)(x)$

d) $\left(\dfrac{g}{h}\right)(x)$

Composition of Functions:

Machine picture:

This combined "machine" is called $f \circ g$ (read "*f* composed with *g*").
The new function $f \circ g$ is defined whenever x is in the domain of g and $g(x)$ is in the domain of f.

Example 2: Suppose $f(x) = 2x^2 - 3x + 1$ and $g(x) = 2x - 4$.

a) $(f \circ g)(x)$

b) $(f \circ g)(-1)$

c) $(g \circ f)(x)$

d) $(g \circ f)(-5)$

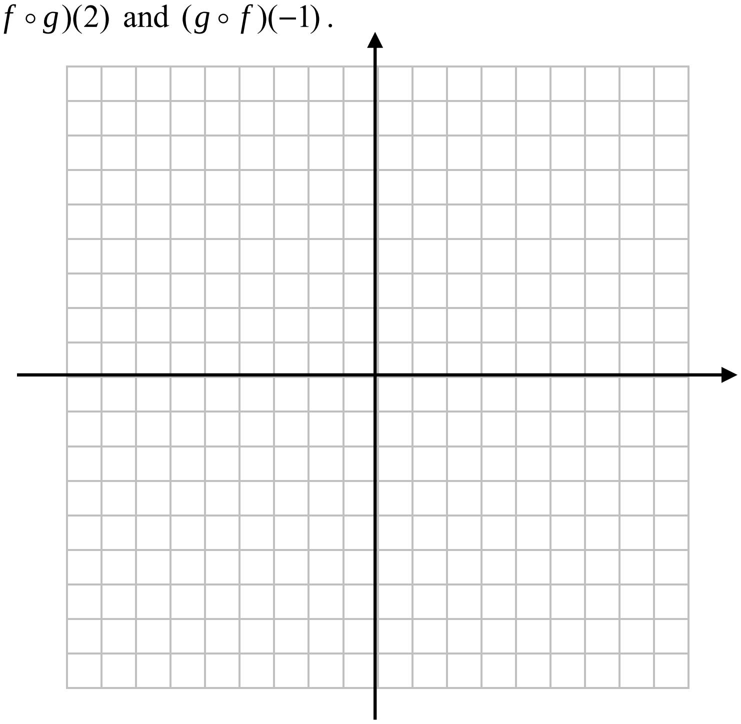
e) $(g \circ g)(x)$

Example 3: Suppose $f(x)=\dfrac{3x-7}{4+8x}$, $g(x)=\dfrac{2}{x+4}$, and $h(x)=2x-5$.

a) $(f\circ h)(x)$

b) $(f\circ g)(x)$

Example 4: Use the given graph to determine $(f\circ g)(2)$ and $(g\circ f)(-1)$.



Finding the domain of compositions:

To find the domain of $f\circ g$, first compose them and find the rule for $f(g(x))$. Then find the domain of this new function $f(g(x))$. Also find the domain of $g(x)$ (the "inside" function). Combine these and you will have the domain of $f\circ g$.

Example 5: Suppose $f(x) = \dfrac{3x}{x-1}$ and $g(x) = x + 5$.

a) Find $f \circ g$ and its domain.

b) Find $g \circ f$ and its domain.

Example 5: Suppose $f(x) = \sqrt{x+5}$ and $g(x) = x^2$.

a) Find $f \circ g$ and its domain.

b) Find $g \circ f$ and its domain.

Example 6: Suppose $f(x)=\dfrac{1}{x}$ and $g(x)=\sqrt{x+4}$. Find $f\circ g$ and its domain.

Finding domain of *f+g*, *f-g*, *fg*, *f/g*:

To be in the domain of these combinations of functions, *x* must be in the domain of both *f* and g.

Example 7: Suppose $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{x-5}$. Find the domain of $f+g$.