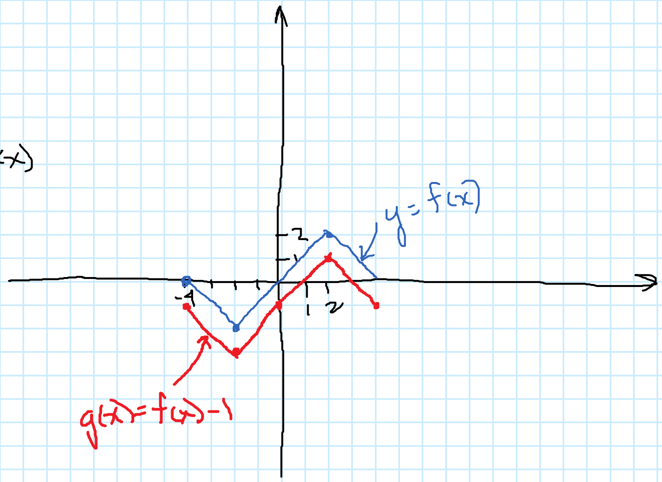
Thursday, October 3, 2019 11:02 AM

2.5 # 17) $g(x) = f(x) - 1$

Start with graph of $y = f(x)$
and shift (translate)
down 1 unit.



2.5 #73

One approach:

$h(x) = \sqrt{-x+2}$

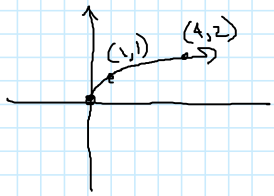
$h(x) = \sqrt{-(x-2)}$

Set $x-2=0$

$x=2$

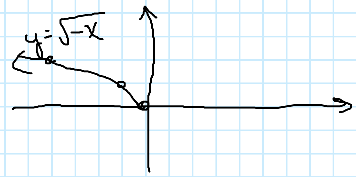
shift it right 2

Parent function: $y=\sqrt{x}$



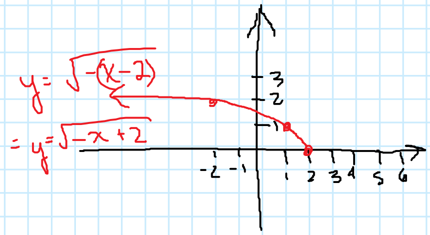
Change to $y=\sqrt{-x}$

Reflects $y=\sqrt{x}$ around y-axis



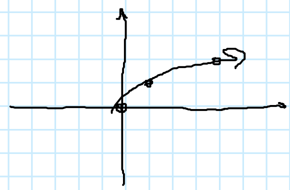
Replace x by $x-2$ to get $y=\sqrt{-(x-2)}$

This shifts it right 2:



Another approach for graphing $h(x)=\sqrt{-x+2}$

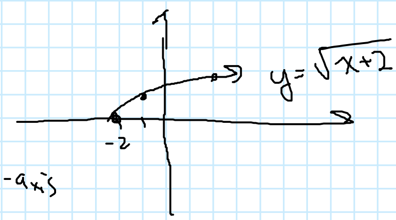
Start with parent function $y=\sqrt{x}$

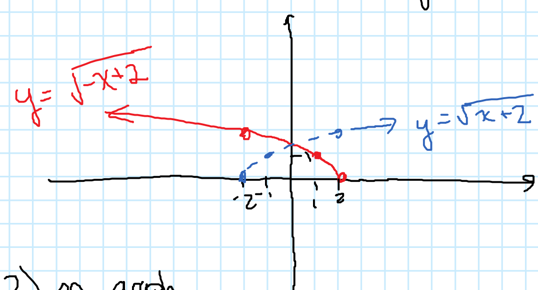


Next graph $y=\sqrt{x+2}$

Shifts graph of $y=\sqrt{x}$ left 2:

Replace x by $-x$: Reflect it around y-axis





Spot-check: Is $(-2,2)$ on graph of $y=\sqrt{-x+2}$?

$2 = \sqrt{-(-2)+2}$

$2 = \sqrt{2+2}$

$2 = \sqrt{4} \Rightarrow 2=2$ True!

The graph of a ~~function~~ equation is the set of ordered pairs that makes the equation true.

1314-3-1-Notes-quadratic-fcns

Thursday, September 26, 2019 10:53 AM

1314-3-1-Notes-quadratic-fcns

3.1: Quadratic Functions

A quadratic function is a function which can be written in the form $f(x) = ax^2 + bx + c$ ($a \neq 0$).

Its graph is a parabola.

Definition: The *maximum* value of a function is the largest y-value on the graph. The *minimum* value of a function is the smallest y-value on the graph.

Standard form for a quadratic function:

Every quadratic function $f(x) = ax^2 + bx + c$ can be written in the form

$$f(x) = a(x-h)^2 + k\ .$$

This is called *standard form* for a quadratic function.

- The vertex of the parabola is (h, k).
- If $a < 0$, the graph opens down and has a *maximum* value.
- If $a > 0$, the graph opens up and has a *minimum* value.
- The larger $|a|$, is the narrower the parabola is.

To write a quadratic function $f(x) = ax^2 + bx + c$ in standard form, we need to complete the square.

Example 1: Sketch the graph of $f(x) = 3x^2 + \overset{18}{\cancel{15}}x - 35$. State the vertex and intercepts. What is the maximum or minimum value?

Complete the square to get it in standard form: $y = a(x-h)^2 + k$

$y = 3x^2 + 18x - 35$

$y = (3x^2 + 18x) - 35$

$y = 3(x^2 + 6x) - 35$

$y = 3(x^2 + 6x + 9) - 35 + -3(9)$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$y = 3(x+3)^2 - 35 - 27$

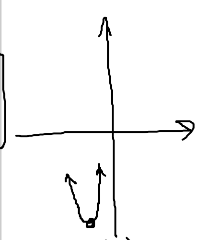
standard form $\boxed{f(x) = 3(x+3)^2 - 62}$

$\boxed{\text{Vertex: } (-3, -62)}$

Find y-intercept: Set $x=0$

$y = f(0) = 3(0)^2 + 18(0) - 35$

$= -35$



Start with $y = x^2$, multiply y-values by 3 (stretch) then shift is left 3, down 62

No maximum value

Ex 1 (cont'd): Find x-intercepts. Set $y=0$. $0 = 3(x+3)^2 - 62$

Ex 1 (cont'd) Find x-intercepts. Set $y=0$.

$0 = 3(x+3)^2 - 62$

$62 = 3(x+3)^2$

$\frac{62}{3} = (x+3)^2$

$20\frac{2}{3} = (x+3)^2$

$20.67 = (x+3)^2$

$(x+3)^2 = 20.67$

$x+3 = \pm\sqrt{20.67}$

$x = -3 \pm \sqrt{20.67}$

$\boxed{\text{No maximum value}}$

$\boxed{\text{Minimum value is } f(-3) = -62}$

3.1.2

Example 2: Sketch the graph of $f(x) = -3x^2 + 15x - 18$. State the vertex and intercepts. What is the maximum or minimum value?

Complete the square to write in standard form.

$f(x) = -3x^2 + 15x - 18$

$f(x) = (-3x^2 + 15x) - 18$

$f(x) = -3(x^2 - 5x) - 18$

$y = -3(x^2 - 5x + \frac{25}{4}) - 18 + \frac{75}{4}$

$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$

(when I wrote $\frac{25}{4}$, I added $-\frac{75}{4}$ to right-hand side)

$y = -3\left(x - \frac{5}{2}\right)^2 - \frac{18}{1}\left(\frac{4}{4}\right) + \frac{75}{4}$

$y = -3\left(x - \frac{5}{2}\right)^2 - \frac{72}{4} + \frac{75}{4}$

$\boxed{f(x) = -3\left(x - \frac{5}{2}\right)^2 + \frac{3}{4}}$ standard form

$\boxed{\text{The maximum value is } f\left(\frac{5}{2}\right) = \frac{3}{4}}$

$\boxed{\text{No minimum value}}$

$\boxed{\text{Vertex of parabola: } \left(\frac{5}{2}, \frac{3}{4}\right)} = \left(2\frac{1}{2}, \frac{3}{4}\right)$

It opens down.

~~**Example 3:** Sketch the graph of $f(x) = -2x^2 - 12x - 16$.~~

Find y-intercept: Set $x=0$

$y = f(0) = -3(0)^2 + 15(0) - 18$

$= -18$

$\boxed{y\text{-intercept: } -18 \text{ or } (0, -18)}$

Find x-intercepts: Set $x=0$

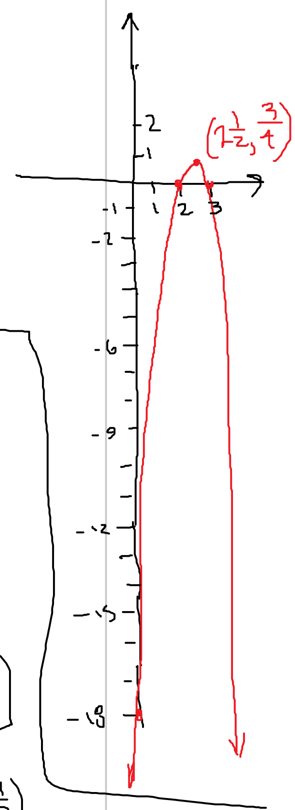
$0 = -3x^2 + 15x - 18$

$0 = -3(x^2 - 5x + 6)$

$0 = -3(x-2)(x-3)$

$x = 2, x = 3$

$\boxed{x\text{-intercepts: } 2 \text{ and } 3 \text{ or } (2,0) \text{ and } (3,0)}$



Example 4: Write $f(x) = x^2 + x + 2$ in standard form. What is its maximum or minimum value? What input produces this maximum or minimum?

$f(x) = x^2 + x + 2$

$y = (x^2 + x) + 2$

$y = (x^2 + 1x + \frac{1}{4}) + 2 + \left(-\frac{1}{4}\right)$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

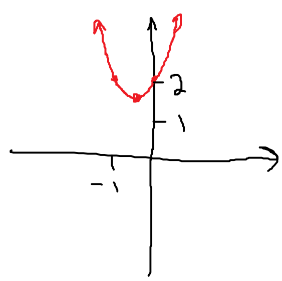
$y = (\quad)^2 + \frac{8}{4} - \frac{1}{4}$

$\boxed{f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{7}{4}}$

Opens up.

Vertex: $\left(-\frac{1}{2}, \frac{7}{4}\right)$

$= \left(-\frac{1}{2}, 1\frac{3}{4}\right)$



$\boxed{\text{Minimum value: } f\left(-\frac{1}{2}\right) = \frac{7}{4}}$

(Minimum value is $\frac{7}{4}$, occurring when $x = -\frac{1}{2}$)

Example 5: Find the maximum or minimum value of $f(x) = -2x^2 - 5x - 1$. (not done during class)

$y = -2(x^2 - 5x) - 1$

$y = -2(x^2 - 5x + \frac{25}{4}) - 1 + \frac{25}{2}$

$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$

when I wrote $\frac{25}{4}$, I added $-2\left(\frac{25}{4}\right) = -\frac{50}{4} = -\frac{25}{2}$

so add $+\frac{25}{2}$ to make up for it.

$y = -2\left(x - \frac{5}{2}\right)^2 - \frac{2}{2} + \frac{25}{2}$

$\boxed{f(x) = -2\left(x - \frac{5}{2}\right) + \frac{23}{2}}$ Opens down. The maximum value is $f\left(\frac{5}{2}\right) = \frac{23}{2}$

Example 6: Find the quadratic function such that $f(3) = -6$ and the vertex is $(-2, -3)$.

$f(x) = a(x-h)^2 + k$

$(h,k) = (-2,-3) \Rightarrow f(x) = a(x+2)^2 - 3$ Need to find a.

$f(3) = -6 \Rightarrow f(3) = a(3+2)^2 - 3 = -6$

$a(5)^2 - 3 = -6$

$25a - 3 = -6$

$+3 \qquad +3$

$25a = -3$

$a = -\frac{3}{25}$

$\boxed{f(x) = -\frac{3}{25}(x+2)^2 - 3}$

Check: $f(3) = -\frac{3}{25}(3+2)^2 - 3 = -\frac{3}{25}(25) - 3$

$= -3 - 3 = -6$ ✓

Tuesday, October 15, 2019 9:49 AM

Example 3: Sketch the graph of $f(x) = -2x^2 - 12x - 16$.

$$f(x) = -2(x^2 + 6x) - 16$$

$$y = -2(x^2 + 6x + \underline{9}) - 16 + \underline{18}$$

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

when I wrote 9, I added $-2(9) = -18$.

So add $+18$ to make up for it.

$$\boxed{f(x) = -2(x+3)^2 + 2}$$

vertex is $(-3, 2)$. Opens down

y-intercept: Set $x = 0 \Rightarrow f(0) = -2(0)^2 - 12(0) - 16 = -16$

x-intercepts. Set $y = 0 \Rightarrow 0 = -2(x+3)^2 + 2$

$$2(x+3)^2 = 2$$

$$\frac{2(x+3)^2}{2} = \frac{2}{2}$$

$$(x+3)^2 = 1$$

$$x + 3 = \pm\sqrt{1}$$

$$x + 3 = \pm 1$$

$$x = -3+1, -3-1$$

$$x = -2, -4$$