

3.1: Quadratic Functions

A quadratic function is a function which can be written in the form $f(x) = ax^2 + bx + c$ ($a \neq 0$).

Its graph is a parabola.

Definition: The *maximum* value of a function is the largest y-value on the graph. The *minimum* value of a function is the smallest y-value on the graph.

Standard form for a quadratic function:

Every quadratic function $f(x) = ax^2 + bx + c$ can be written in the form

$$f(x) = a(x - h)^2 + k.$$

This is called *standard form* for a quadratic function.

- The vertex of the parabola is (h, k) .
- If $a < 0$, the graph opens down and has a *maximum* value.
- If $a > 0$, the graph opens up and has a *minimum* value.
- The larger $|a|$, is the narrower the parabola is.

To write a quadratic function $f(x) = ax^2 + bx + c$ in standard form, we need to complete the square.

Example 1: Sketch the graph of $f(x) = 3x^2 + 16x - 35$. State the vertex and intercepts. What is the maximum or minimum value?

Example 2: Sketch the graph of $f(x) = -3x^2 + 15x - 18$. State the vertex and intercepts. What is the maximum or minimum value?

Example 3: Sketch the graph of $f(x) = -2x^2 - 12x - 16$.

Example 4: Write $f(x) = x^2 + x + 2$ in standard form. What is its maximum or minimum value? What input produces this maximum or minimum?

Example 5: Find the maximum or minimum value of $f(x) = -2x^2 - 5x - 1$.

Example 6: Find the quadratic function such that $f(3) = -6$ and the vertex is $(-2, -3)$.