1314-3-2-Notes-polynomials

Monday, November 18, 2019 12:34 PM

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3.2: Polynomial Functions and Their Graphs

Definition: A polynomial function is a function which can be written in the form

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, where n is any positive integer

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

Example 1:

Examples of polynomials: $f(x) = 3x^4 + 7x^2 - 8$

$g(x) = -\frac{1}{2}x^7 - 12x^3 + 3x^2 - \pi x$

Not polynomials: $h(x) = 3x^{-4} + 5x^{-1} \longleftarrow$ Rewrite as $\frac{3}{x^4} + \frac{5}{x}$

$j(x) = 2x^{\frac{1}{2}} - 5x^{\frac{1}{3}} \longleftarrow$ Rewrite as $2\sqrt{x} - 5\sqrt[3]{x}$

The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the coefficients of the polynomial function.

Note: The variable is only raised to positive integer powers–no negative or fractional exponents. However, the coefficients may be any real numbers, including fractions or irrational numbers like π or $\sqrt{7}$.

The degree of the polynomial is the largest exponent on x. (Degree is usually denoted by n.)

The leading coefficient of a polynomial is the coefficient of the term with the largest power of x.

Example 2: $f(x) = 2x^2 - 9x^4 + 7x^3 - 12$ Could rewrite: $f(x) = -9x^4 + 7x^3 + 2x^2 - 12$

Degree: 4

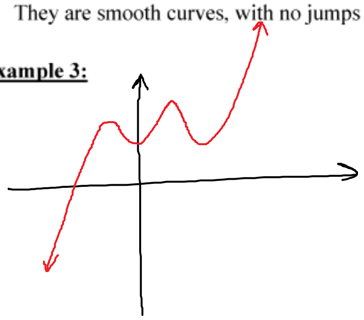
Leading coefficient: -9

Leading term: $-9x^4$

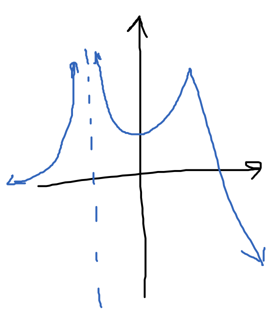
Facts about polynomials:

- They are smooth curves, with no jumps or sharp points.

Example 3:



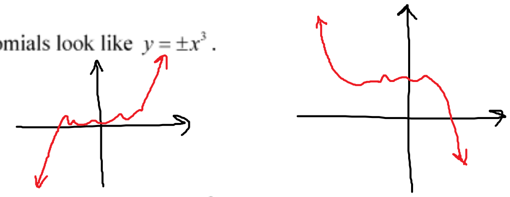
likely a polynomial



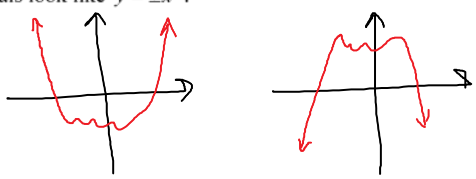
Not a polynomial

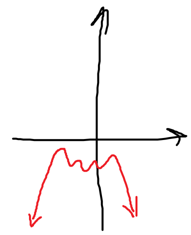
- A polynomial has at most $n-1$ turning points.
- A polynomial has at most n x-intercepts.
- A polynomial has exactly one y-intercept.
- Every polynomial has domain $(-\infty, \infty)$.
- Near the ends,

  Odd-degree polynomials look like $y = \pm x^3$.



  Even-degree polynomials look like $y = \pm x^2$.

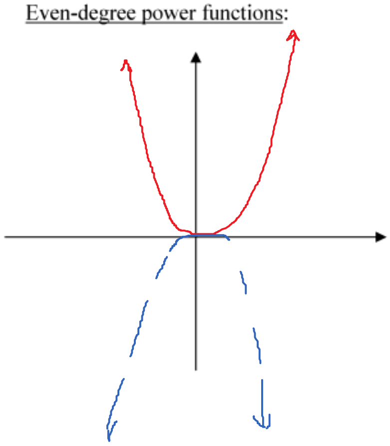




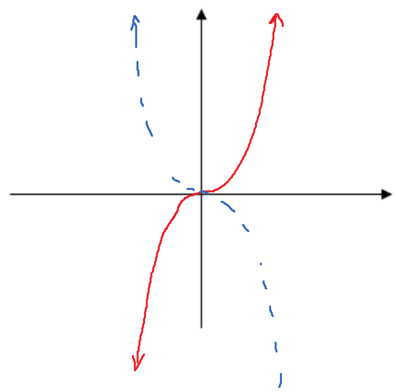
Power functions:

A *power function* is generally defined to be a polynomial which takes the form $f(x) = ax^n$, where n is a positive integer. Modifications of power functions can be graphed using transformations.

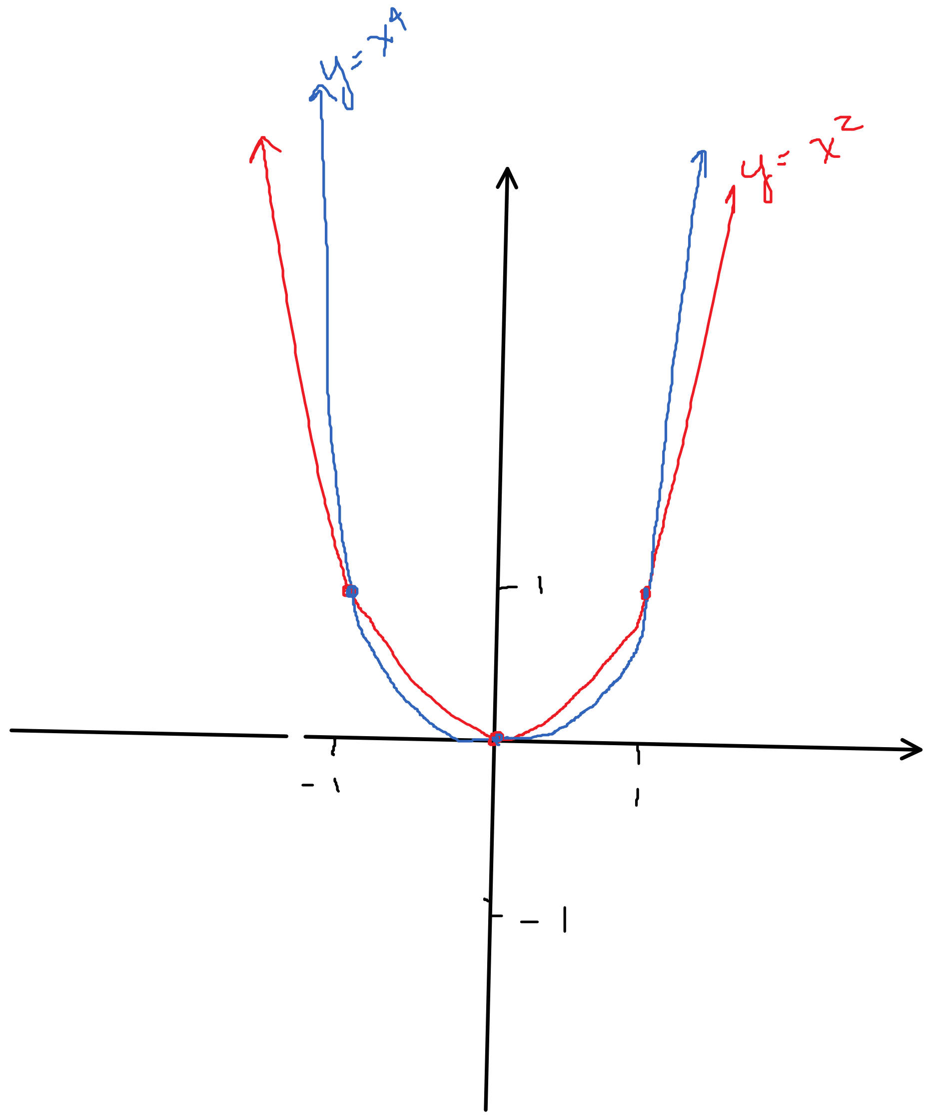
Even-degree power functions:

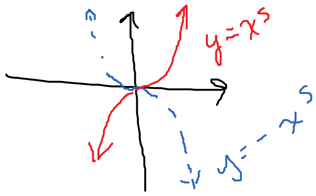


Odd-degree power functions:



$y = x^4$
still has
(0,0)
(1, 1)
(-1,1)

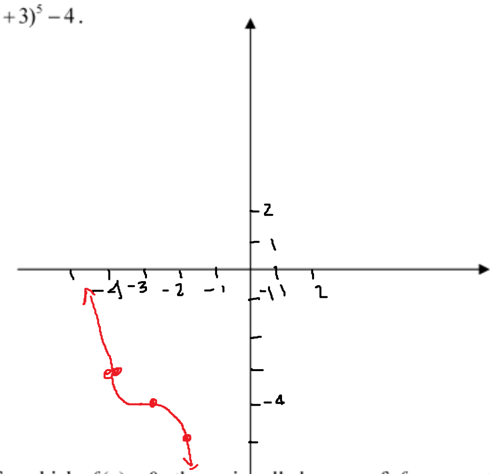




Note: Multiplying any function by a will multiply all the y-values by a. The general shape will stay the same.

Example 4: Sketch the graph of $y=-(x+3)^5-4$.

Parent function
$y=x^5$, similar to $y=x^3$ except steeper on sides and flatter in middle section
reflect around x-axis
shift it left 3, down 4



Zeros of polynomials:

If f is a polynomial and c is a real number for which $f(c)=0$, then c is called a *zero* of f, or a *root* of f.

Ex: $f(x) = y = x^2 - x - 2$

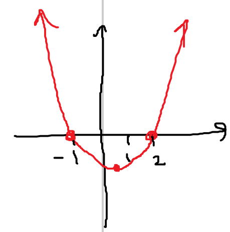
To find x-intercepts, set $y=0$:

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x-2=0 \quad | \quad x+1=0$

$x=2 \quad | \quad x=-1$



If c is a zero of f, then

- c is an x-intercept of the graph of f.
- $(x-c)$ is a factor of f.

Could write:

$f(x) = x^2 - x - 2$

$f(x) = (x-2)(x+1)$

So if we have a polynomial in factored form, we know all of its x-intercepts.

- every factor gives us an x-intercept.
- every x-intercept gives us a factor.

Example 5: Consider the function $f(x)=3x(x-3)^6(2x-1)^3(x+2)^2$.

$3x=0 \quad \frac{3x}{3}=\frac{0}{3} \quad x=0$ | $x-3=0 \quad x=3$ | $2x-1=0 \quad 2x=1 \quad x=\frac{1}{2}$ | $x+2=0 \quad x=-2$

Zeros (x-intercepts):

$\boxed{\text{Zeros: } 0, 3, \tfrac{1}{2}, -2}$

0 is a zero of multiplicity 1
3 is a zero of multiplicity 6
$\frac{1}{2}$ is a zero of multiplicity 3
-2 is a zero of multiplicity 2

To get the degree, add the multiplicities of all the factors:

$\underbrace{1+6+3+2}_{\text{exponents}} = 12 \Rightarrow \boxed{12^{th} \text{ degree}}$

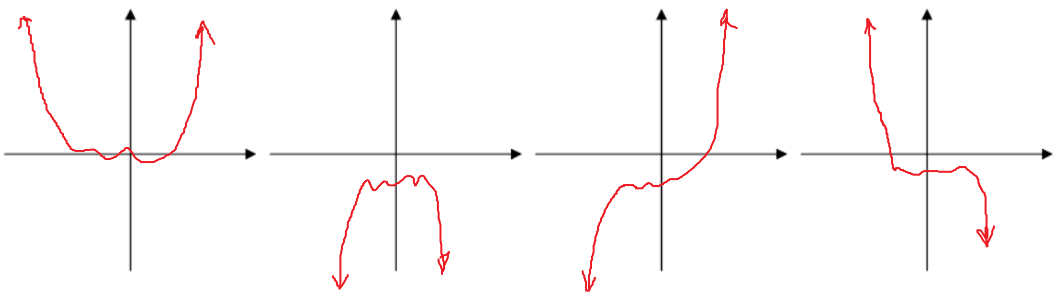
The leading term is:

$3x(x)^6(2x)^3(x)^2 = 3x \cdot x^6 \cdot 8x^3 \cdot x^2 = \boxed{24x^{12}}$

Take the leading term of each factor and raise it to the power that's on that factor.

<u>Steps to graphing other polynomials</u>:

1. Factor and find x-intercepts.
2. Mark x-intercepts on x-axis.
3. Determine the leading term.
    - Degree: is it odd or even?
    - Sign: is the coefficient positive or negative?
4. Determine the end behavior. What does it "look like"?



Even Degree Sign (+) | Even Degree Sign (-) | Odd Degree Sign (+) | Odd Degree Sign (-)

5. For each x-intercept, determine the behavior.
    - <u>Even multiplicity</u>: touches x-axis, but doesn't cross (looks like a parabola there).
    - <u>Odd multiplicity of 1</u>: crosses the x-axis (looks like a line there).
    - <u>Odd multiplicity</u> ≥ 3 : crosses the x-axis and looks like a cubic there.

<u>Note</u>: It helps to make a table as shown in the examples below.

6. Draw the graph, being careful to make a nice smooth curve with no sharp corners.

<u>Note</u>: without calculus or plotting lots of points, we don't have enough information to know how high or how low the turning points are.

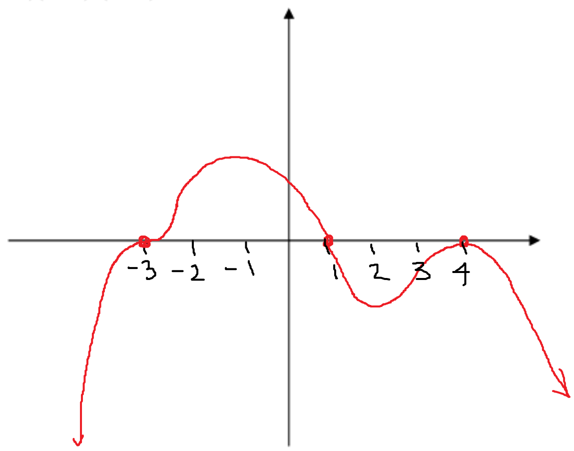
Example 6: Sketch the graph of $g(x) = -(x-1)(x+3)^3(x-4)^2$.

Zeros (x-intercepts):
1, −3, 4

Zeros	multiplicity	Looks like
1	1	line
−3	3	cubic
4	2	parabola

Sum = 6

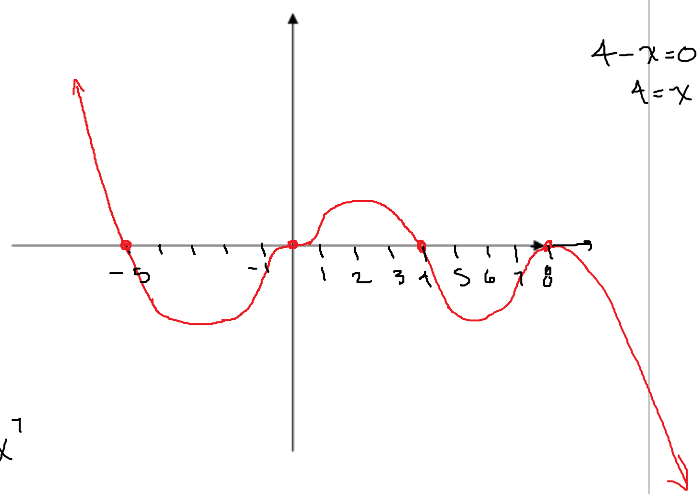
degree: $n = 6$



degree: 6 even, Sign on leading term: Neg

Example 7: Sketch the graph of $f(x) = x^3(4-x)(x+5)(x-8)^2$.

Zeros	Mult.	Looks Like
0	3	cubic
4	1	line
−5	1	line
8	2	parabola



$4 - x = 0$
$4 = x$

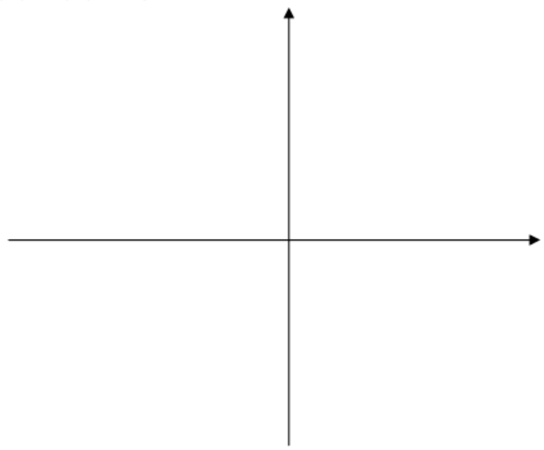
Leading term: $x^3(-x)(x)(x)^2 = -x^7$

Sign: Neg

Degree: 7 odd

End behavior:

Example 8: Sketch the graph of $y = 2(x+1)^5(x+7)^2(2x-7)$.



Example 9: Sketch the graph of $P(x) = x^3 + 3x^2 - 4x - 12$.

Need to factor it.

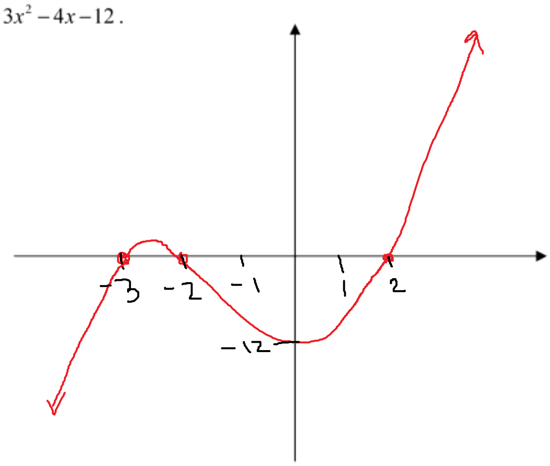
$P(x) = (x^3 + 3x^2) + (-4x - 12)$

$= x^2(x+3) - 4(x+3)$

$= (x+3)(x^2 - 4)$

$P(x) = (x+3)(x+2)(x-2)$

Zeros	mult	Looks Like
-3	1	line
-2	1	line
2	1	line



Leading term: x^3

Sign: +

Degree: 3 odd

Ends:

Skip

Intermediate Value Theorem for Polynomials

Let f be a polynomial function with real coefficients. If $f(a)$ and $f(b)$ have opposite signs, then there is at least one value of c between a and b for which $f(c)=0$.

Example 1: Show that $f(x)=3x^3-10x+9$ has a real zero between -3 and -2.