1314-3-3-Notes-poly-division

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3.3: Dividing Polynomials; Remainder and Factor Theorems

Terms you should know:

$$\text{Divisor}\overline{\big)\text{Dividend}}^{\ \text{Quotient}}$$

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Remainder

Recall: Dividend = (Divisor)(Quotient) + Remainder

So… $\dfrac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}}$

Example 1: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ -36 \\ \hline 19 \\ 12 \\ \hline 7 \end{array}$$

$$\frac{379}{12} = 379 \div 12 = \boxed{31 + \frac{7}{12}}$$

Long division of polynomials will be done similarly…

Example 2: Divide $\frac{3x^2+7x+9}{x+2}$.

$$\begin{array}{r} 3x + 1 \\ x+2\overline{)3x^2 + 7x + 9} \\ +(-3x^2 + 6x) \\ \hline x + 9 \\ +(-x + 2) \\ \hline 7 \end{array}$$

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1 + \frac{7}{x+2}}$$

> ***Important:***
>
> You *must* put zeros in place of any missing powers of x.
> The 0's act as "placeholders" just as they do in our number system.
>
> For example, 206 is certainly not the same number as 26. The 0 "holds" the 10's place.
>
> Similarly, when doing division, we write $2x^2+0x+6$ instead of $2x^2+6$.

Example 3: Divide $\dfrac{8x^4-10x^2+3x-2}{2x-1}$.

$\frac{8x^4}{2x}$ $\frac{4x^3}{2x}$ $\frac{-8x^2}{2x}$ $\frac{-x}{2x}$

$$
\begin{array}{r|l}
 & 4x^3 + 2x^2 - 4x - \frac{1}{2} \\
2x-1 & 8x^4 + 0x^3 - 10x^2 + 3x - 2 \\
 & \underline{-(8x^4 - 4x^3)} \\
 & 4x^3 - 10x^2 \\
 & \underline{-(4x^3 - 2x^2)} \\
 & -8x^2 + 3x \\
 & \underline{-(-8x^2 + 4x)} \\
 & -x - 2 \\
 & \underline{-(-x + \frac{1}{2})} \\
\end{array}
$$

$-2-\frac{1}{2}=-\frac{5}{2}=-2\frac{1}{2}$

$2x(?)=-x$

$\frac{-x}{2x}=-\frac{1}{2}$

$$\frac{8x^4-10x^2+3x-2}{2x-1}=\boxed{4x^3+2x^2-4x-\frac{1}{2}+\frac{-5/2}{2x-1}}$$

Check. $(2x-1)\left(4x^3+2x^2-4x-\frac{1}{2}\right)-\frac{5}{2}=8x^4-10x^2+3x-2$

Example 4: Divide $\dfrac{x^4-5x^3+6x-8}{x^2+2}$.

Example 5: Divide $\frac{x^3 - 2x^2 - 5x + 6}{x - 3}$.

You can use this long division to factor $x^3 - 2x^2 - 5x + 6$ …

The Factor Theorem

Let $f(x)$ be a polynomial,

- If $f(c) = 0$, then $x - c$ is a factor of $f(x)$. Note: c can be negative
- If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

In other words, c is a zero (root, x-intercept) of $f(x)$ if and only if $x - c$ is a factor of $f(x)$.

Example 6: Show that 4 is a zero of the function $P(x) = x^3 + 3x^2 - 18x - 40$. Use this fact to factor P completely and find all other zeros of $P(x)$.

$P(-4) = (-4)^3 + 3(-4)^2 - 18(-4) - 40$
$= -64 + 3(16) + 72 - 40$
$= -64 + 48 + 32$
$\neq 0$

$P(4) = (4)^3 + 3(4)^2 - 18(4) - 40$
$= 64 + 48 - 72 - 40$
$= 112 - 112 = 0$ ✓

So 4 is a zero.
$x - 4$ is a factor

$$\begin{array}{r|rrrr} 4 & 1 & 3 & -18 & -40 \\ & & 4 & 28 & 40 \\ \hline & 1 & 7 & 10 & \boxed{0} \end{array}$$

$\Rightarrow \frac{x^3 + 3x^2 - 18x - 40}{x - 4} = x^2 + 7x + 10$

$P(x) = x^3 + 3x^2 - 18x - 40 = (x-4)(x^2 + 7x + 10)$

$\boxed{P(x) = (x-4)(x+2)(x+5)}$

$\boxed{\text{Zeros of } P\text{: } 4, -2, -5}$

Synthetic Division: This is a "shortcut" which can be used to divide a polynomial by $x-c$, where c is a real number. Synthetic division does *not* work for divisors like x^2+1, x^2+2x-7, etc.

Example 7: Divide $\dfrac{3x^3+4x^2-13}{x-2}$.

Do it with long division first:

$$\begin{array}{r} 3x^2 + 10x + 20 \\ x-2\,\overline{)\,3x^3 + 4x^2 + 0x - 13} \\ -(3x^3 - 6x^2) \\ \hline 10x^2 + 0x \\ -(10x^2 - 20x) \\ \hline 20x - 13 \\ -(20x - 40) \\ \hline 27 \end{array}$$

If $x-2$ is a factor, then 2 is a zero

With synthetic division, the above work can be reduced to:

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & 40 \\ \hline & 3 & 10 & 20 & 27 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & 40 \\ \hline & 3 & 10 & 20 & \boxed{27} \end{array}$$

$$\frac{3x^3+4x^2-13}{x-2} = \boxed{3x^2+10x+20+\frac{27}{x-2}}$$

How to do synthetic division:

Step 1: Write the dividend in descending powers. Then, copy the coefficients and insert 0 for any missing powers.

$$3 \quad 4 \quad 0 \quad -13$$

Step 2: Write the usual division symbol over the numbers. If the divisor is $x-c$, write "c" to the left. Leave space under the numbers and draw a horizontal line.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & & & \\ \hline & & & & \end{array}$$

Step 3: Bring down the first number under the division sign.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & & & \\ \hline & 3 & & & \end{array}$$

Step 4: Multiply the latest entry of Row 3 by the number "outside" and write that product on Row 2 in the next column.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & & \\ \hline & 3 & & & \end{array}$$

Step 5: Add the numbers in the new column and write the sum on Row 3.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & & \\ \hline & 3 & 10 & & \end{array}$$

Step 6: Repeat Step 4 and Step 5 until all the columns have been used.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & \\ \hline & 3 & 10 & 20 & \end{array} \qquad \begin{array}{r|rrrr} 2 & 3 & 4 & 0 & -13 \\ & & 6 & 20 & 40 \\ \hline & 3 & 10 & 20 & 27 \end{array}$$

Step 7: The rightmost number on Row 3 is the remainder. The remaining entries on Row 3 are the coefficients of the quotient. The degree of the quotient is one less than the degree of the dividend.

Remainder: 27

Quotient: $3x^2+10x+20$

Therefore,

$$3x^3+4x^2-13=(x-2)(3x^2+10x+20)+\frac{27}{x-2}$$

Example 8: Divide $\frac{x^3-2x^2-5x+6}{x-3}$.

Example 9: Divide $\frac{x^4+7x^3+8x^2-28x-40}{x+3}$.

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 8 & -28 & -40 \\ & & -3 & -12 & 12 & 48 \\ \hline & 1 & 4 & -4 & -16 & \boxed{8} \end{array}$$

Note: If $x+3$ were a factor, then -3 would be a zero.

$$\frac{x^4+7x^3+8x^2-28x-40}{x+3}$$

$$= (x^4+7x^3+8x^2-28x-40) \div (x+3)$$

$$= \boxed{x^3+4x^2-4x-16+\frac{8}{x+3}}$$

To check it:

$$x^4+7x^3+8x^2-28x-40$$

$$= (x+3)(x^3+4x^2-4x-16)+8$$

Example 10: Divide $\frac{x^3+3x^2-18x-40}{x-4}$.

Example 11: Show that 3 is a zero of the function $P(x) = 2x^3 + 7x^2 - 19x - 60$. Use this fact to factor P completely and find all other zeros of $P(x)$.

> ***Remainder Theorem:***
>
> If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Example 12: Given $f(x) = x^4 + 9x^3 + 5x^2 - 12x - 13$, use the Remainder Theorem to find $f(-4)$.

$$\begin{array}{r|rrrrr} -4 & 1 & 9 & 5 & -12 & -13 \\ & & -4 & -20 & 60 & -192 \\ \hline & 1 & 5 & -15 & 48 & -205 \end{array}$$

$$\boxed{f(-4) = -205}$$

$$f(x) = x^4 + 9x^3 + 5x^2 - 12x - 13$$
$$= (x+4)(x^3 + 5x^2 - 15x + 48) - 205$$

Example 13: Solve the equation $x^4 + 7x^3 + 8x^2 - 28x - 48 = 0$ given that -3 is a zero of $P(x) = x^4 + 7x^3 + 8x^2 - 28x - 48$.

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 8 & -28 & -48 \\ & & -3 & -12 & 12 & 48 \\ \hline & 1 & 4 & -4 & -16 & 0 \end{array}$$

$$P(x) = x^4 + 7x^3 + 8x^2 - 28x - 48$$
$$= (x+3)(x^3 + 4x^2 - 4x - 16)$$
$$= (x+3)\left[(x^3 + 4x^2) + (-4x - 16)\right]$$
$$= (x+3)\left[x^2(x+4) - 4(x+4)\right]$$
$$= (x+3)\left[(x+4)(x^2 - 4)\right]$$

$$(x+3)(x+4)(x+2)(x-2) = 0$$
$$x = -3, -4, -2, 2$$
$$\boxed{\text{Sol'n Set: } \{-3, -4, \pm 2\}}$$

Example: Divide $\dfrac{6x^3-19x^2-65x+50}{2x+5}$.

Step 1: Divide $\dfrac{6x^3}{2x}=$

$$\begin{array}{rl} & 3x^2 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \end{array}$$

Step 2: Multiply $3x^2$ by $2x+5$

$$\begin{array}{rl} & 3x^2 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{6x^3+15x^2} \end{array}$$

Step 3: Subtract (Don't forget the parentheses!)

$$\begin{array}{rl} & 3x^2 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2 \end{array}$$

Step 4: Bring Down

$$\begin{array}{rl} & 3x^2 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \end{array}$$

Repeat the steps…

Step 1: Divide $\dfrac{-34x^2}{2x}=$

$$\begin{array}{rl} & 3x^2-17x \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \end{array}$$

Step 2: Multiply $-17x$ by $2x+5$

$$\begin{array}{rl} & 3x^2-17x \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \\ & \quad -34x^2-85x \end{array}$$

Go to the top right…

Step 4: Bring Down

$$\begin{array}{rl} & 3x^2-17x \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \\ & \quad \underline{-(-34x^2-85x)} \\ & \qquad\qquad 20x+50 \end{array}$$

Repeat the steps…

Step 1: Divide $\dfrac{20x}{2x}=$

$$\begin{array}{rl} & 3x^2-17x+10 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \\ & \quad \underline{-(-34x^2-85x)} \\ & \qquad\qquad 20x+50 \end{array}$$

Step 2: Multiply 10 by $2x+5$

$$\begin{array}{rl} & 3x^2-17x+10 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \\ & \quad \underline{-(-34x^2-85x)} \\ & \qquad\qquad 20x+50 \\ & \qquad\qquad 20x+50 \end{array}$$

Step 3: Subtract (Don't forget the parentheses!)

$$\begin{array}{rl} & 3x^2-17x+10 \\ \cline{2-2} 2x+5\big) & 6x^3-19x^2-65x+50 \\ & \underline{-(6x^3+15x^2)} \\ & \quad -34x^2-65x \\ & \quad \underline{-(-34x^2-85x)} \\ & \qquad\qquad 20x+50 \\ & \qquad\quad \underline{-(20x+50)} \\ & \qquad \text{Remainder} \rightarrow 0 \end{array}$$

Extra Handout on Polynomial Long Division