1314-3-4-rational-zeros

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Recall:
A rational number: can be written as the ratio (fraction, quotient of two integers)

3.4: Zeros of Polynomial Functions

The Rational Zero Theorem: (Rational Roots Theorem)

If a polynomial has integer coefficients, and if $\frac{p}{q}$ is a rational zero in reduced form, then p is a factor of the constant term, and q is a factor of the leading coefficient.

Example 1: Consider $f(x) = 6x^2 - 19x - 7$. Find the ~~possible rational zeros~~ and the actual zeros.

$$f(x) = 6x^2 - 19x - 7$$
$$= (3x + 1)(2x - 7)$$

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$\wedge$
$1 \cdot 42$
$2 \cdot 21$
$3 \cdot 14$
$6 \cdot 7$

If we had $f(x) = 0$, or

$3x + 1 = 0$
$3x = -1$
$x = -\frac{1}{3}$

$2x - 7 = 0$
$2x = 7$
$x = \frac{7}{2}$

Actual $\boxed{\text{zeros are: } -\frac{1}{3}, \frac{7}{2}}$

Example 2: Find the possible rational zeros of $g(x) = 4x^4 - 5x^3 + 3x^2 - 13x - 18$.

Factors of constant term 18: 1, 18, 2, 9, 3, 6

Factors of leading coefficient 4: 1, 4, 2

Possible rational zeros: $\pm \left\{ \frac{1}{1}, \frac{1}{4}, \frac{1}{2}, \frac{18}{1}, \frac{18}{4}, \frac{18}{2}, \frac{2}{1}, \frac{2}{4}, \frac{2}{2}, \frac{9}{1}, \frac{9}{4}, \frac{9}{2}, \frac{3}{1}, \frac{3}{4}, \frac{3}{2}, \frac{6}{1}, \frac{6}{4}, \frac{6}{2} \right\}$

$= \pm \left\{ 1, \frac{1}{4}, \frac{1}{2}, 18, \frac{9}{2}, 9, 2, \frac{9}{4}, 3, \frac{3}{4}, \frac{3}{2}, 6 \right\}$

To find zeros of a polynomial function:

- List the possible rational zeros.
- Then use synthetic division to find one that gives a zero remainder and is therefore a zero.
- Use the result of your synthetic division to factor (partially) the polynomial.
- Repeat the process until the polynomial is completely factored.

Example 3: Find the zeros of $f(x) = 2x^3 + 5x^2 - 22x + 15$.

factors of
constant term: $\pm 1, 15, 3, 5$

Factors of lead. coeff 2: $\pm 1, 2$

Possible rational zeros: $\pm\left\{\frac{1}{1}, \frac{1}{2}, \frac{15}{1}, \frac{15}{2}, \frac{3}{1}, \frac{3}{2}, \frac{5}{1}, \frac{5}{2}\right\} \Rightarrow \pm\left\{1, \frac{1}{2}, 15, \frac{15}{2}, 3, \frac{3}{2}, 5, \frac{5}{2}\right\}$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -22 & 15 \\ & & 2 & 7 & -15 \\ \hline & 2 & 7 & -15 & \boxed{0} \end{array}$$

$f(x) = 2x^3 + 5x^2 - 22x + 15$
$= (x-1)(2x^2 + 7x - 15)$
$= (x-1)(2x - 3)(x + 5)$

$x - 1 = 0$, $x = 1$ | $2x - 3 = 0$, $2x = 3$, $x = \frac{3}{2}$ | $x + 5 = 0$, $x = -5$

$\boxed{\text{Zeros: } 1, \frac{3}{2}, -5}$

$2(15) = 30$
$1 \cdot 30$
$2 \cdot 15$
$3 \cdot 10$
$5 \cdot 6$

Example 4: Find the zeros of $f(x) = 3x^3 + 8x^2 - 7x - 12$.

Example 5: Solve $x^3 + 11x^2 + 26x - 8 = 0$.

Example 6: Solve $x^4 - 6x^3 + x^2 + 24x - 20 = 0$.

Factors of constant term: 1, 20, 2, 10, 4, 5

Factors of lead. coeff 1: 1

Possible rational zeros: $\pm \left\{ \frac{1}{1}, \frac{20}{1}, \frac{2}{1}, \dots \right\} \Rightarrow \pm \{1, 20, 2, 10, 4, 5\}$

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 1 & 24 & -20 \\ & & 1 & -5 & -4 & 20 \\ \hline & 1 & -5 & -4 & 20 & \boxed{0} \end{array}$$

$x^4 - 6x^3 + x^2 + 24x - 20 = 0$

$(x-1)(x^3 - 5x^2 - 4x + 20) = 0$

$(x-1)[x^2(x-5) - 4(x-5)] = 0$

$(x-1)(x-5)(x^2-4) = 0$

$(x-1)(x-5)(x+2)(x-2) = 0$

$x = 1, 5, -2, 2$

Sol'n Set: $\boxed{\{1, 5, -2, 2\}}$

Example 7: Solve $x^4 + x^3 - 4x^2 - 16x - 24 = 0$.

Possible rational zeros: $\pm\{\cancel{1}, 24, 2, 12, 3, 8, 4, 6\}$

$$\begin{array}{r|rrrrr} -1 & 1 & 1 & -4 & -16 & -24 \\ & & -1 & 0 & 4 & 12 \\ \hline & 1 & 0 & -4 & -12 & \boxed{-12} \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & -4 & -16 & -24 \\ & & -2 & 2 & 4 & 24 \\ \hline & 1 & -1 & -2 & -12 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -2 & -12 \\ & & 2 & 2 & 0 \\ \hline & 1 & 1 & 0 & \boxed{-12} \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -2 & -12 \\ & & 3 & 6 & 12 \\ \hline & 1 & 2 & 4 & \boxed{0} \end{array}$$

$x^4 + x^3 - 4x^2 - 16x - 24 = 0$

$(x+2)(x^3 - x^2 - 2x - 12) = 0$

$(x+2)(x-3)(x^2 + 2x + 4) = 0$

$x+2=0$ | $x-3=0$ | $x^2 + 2x + 4 = 0$

$x=-2$ | $x=3$ | $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2 \pm i\sqrt{4 \cdot 3}}{2} = \dfrac{-2 \pm 2i\sqrt{3}}{2}$

$= \dfrac{\cancel{2}(-1 \pm i\sqrt{3})}{\cancel{2}} = -1 \pm i\sqrt{3}$ $\Rightarrow$ $\boxed{\text{Sol'n Set: } \{-2, 3, -1 \pm i\sqrt{3}\}}$

Important Facts:

- A polynomial of degree n has n real or complex roots, counting multiplicities. So it can be written as a product of n factors.
- If a non-real number is the root of a polynomial, then its complex conjugate is also a root.

Example 8: $2i$ is a root of the equation $x^4 - 9x^3 + 22x^2 - 36x + 72 = 0$. Find the solution set.

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If $2i$ is a root, $-2i$ is also a root

$(x - 2i)$ and $(x + 2i)$ are factors.

$(x - 2i)(x + 2i)$

$= x^2 + \cancel{2ix} - \cancel{2ix} - 4i^2$

$= x^2 - 4(-1)$

$= x^2 + 4$

Note. $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

To find the other solutions, divide: $x^2 + 4 \overline{\big) x^4 - 9x^3 + 22x^2 - 36x + 72}$

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Example 9: $3-i$ is a root of $x^3-10x^2+34x-40=0$. Solve the equation.

If $3-i$ is a root, then $3+i$ is also a root.

Example 10: Factor the polynomial over the a) rational numbers; b) real numbers; c) complex numbers.

x^4+4x^2-45

Example 11: Find a polynomial of degree 3 that has zeros 3 and $-2i$ and has $f(-1)=40$.