3.4: Zeros of Polynomial Functions

The Rational Zero Theorem:

If a polynomial has integer coefficients, and if $\frac{p}{q}$ is a rational zero in reduced form, then p is a factor of the constant term, and q is a factor of the leading coefficient.

Example 1: Consider $f(x) = 6x^2 - 19x - 7$. Find the possible rational zeros and the actual zeros.

Example 2: Find the possible rational zeros of $g(x) = 4x^4 - 5x^3 + 3x^2 - 13x - 18$.

To find zeros of a polynomial function:

- List the possible rational zeros.
- Then use synthetic division to find one that gives a zero remainder and is therefore a zero.
- Use the result of your synthetic division to factor (partially) the polynomial.
- Repeat the process until the polynomial is completely factored.

Example 3: Find the zeros of $f(x) = 2x^3 + 5x^2 - 22x + 15$.

Example 4: Find the zeros of $f(x) = 3x^3 + 8x^2 - 7x - 12$.

Example 5: Solve $x^3 + 11x^2 + 26x - 8 = 0$.

Example 6: Solve $x^4 - 6x^3 + x^2 + 24x - 20 = 0$.

Example 7: Solve $x^4 + x^3 - 4x^2 - 16x - 24 = 0$.

Important Facts:

- A polynomial of degree *n* has *n* real or complex roots, counting multiplicities. So it can be written as a product of *n* factors.
- If a non-real number is the root of a polynomial, then its complex conjugate is also a root.

Example 8: $2i$ is a root of the equation $x^4 - 9x^3 + 22x^2 - 36x + 72$. Find the solution set.

Example 9: $3-i$ is a root of $x^3-10x^2+34x-40=0$. Solve the equation.

Example 10: Factor the polynomial over the a) rational numbers; b) real numbers; c) complex numbers.

x^4+4x^2-45

Example 11: Find a polynomial of degree 3 that has zeros 3 and $-2i$ and has $f(-1)=40$.