3.6: Polynomial and Rational Inequalities

Polynomial and rational inequalities are examples of *nonlinear* inequalities. They cannot be solved simply by adding, subtracting, multiplying, or dividing both sides by the same quantity.

Examples: $\frac{x-5}{x+3} \geq 0$ or $x^2 - 7x + 12 > 0$

Can we do this?

$$x^2 - 7x + 12 > 0$$
$$(x-3)(x-4) > 0$$
$$x - 3 > 0 \text{ or } x - 4 > 0$$
$$x > 3 \text{ or } x > 4$$

Does this give us the right answer?

If not, where did our logic go wrong?

To solve a polynomial or rational inequality:

- Rearrange so that one side is 0.
- If the nonzero side involves quotients, write it with a common denominator.
- If possible, factor the nonzero side. If it's a quotient, factor the numerator and factor the denominator.
- Find all the numbers that make the expression zero or undefined.
- Use these values to divide the number line into intervals.
- In each interval, choose a test number to determine whether the expression is positive or negative. It's easiest to use the factored form.
- Use this information to make a "sign chart".
- Use the sign chart to determine what values make the original inequality true.
- Write the solution in interval notation.

Example 1: Solve $9 - x^2 \le 0$.

Example 2: Solve $x^2 - 7x > -12$.

Example 3: Solve $-2(x+5)(x-1)(x-4)^2 \ge 0$

Example 4: Solve $3x^2 - 7x < 6$.

Example 5: Solve $\frac{2x+6}{x-2} \geq 0$.

Example 6: Solve $\frac{3+x}{3-x} \geq 1$.

Example 7: Solve $\frac{1}{x-2} < \frac{2}{x+2}$.