1314-4-1-Notes-exponentials

Thursday, October 17, 2019 11:21 AM

1314-4-1-Notes-exponentials

4.1: Exponential Functions

An *exponential* function takes the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$.

Example 1: Examples of exponential functions:

$f(x) = 5^x$, $g(x) = \left(\frac{2}{3}\right)^x$

Why must we have $b > 0$ and $b \neq 1$?

Why $b > 0$? Suppose $b = -4$: $y = (-4)^x$

$x = \frac{1}{2} \Rightarrow y = (-4)^{1/2} = \sqrt{-4}$ not a real number!

Why $b \neq 1$? $b = 1 \Rightarrow f(x) = b^x = 1^x$

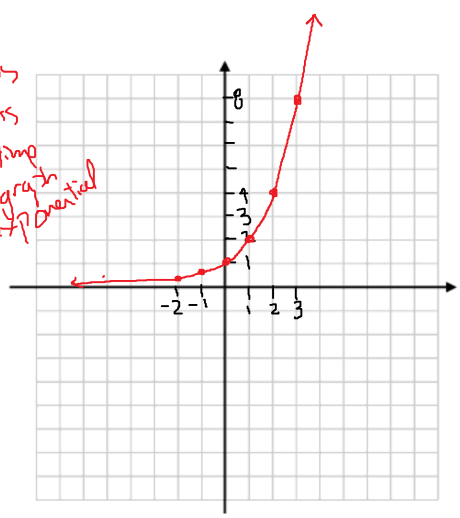
$f(x) = 1$ constant function

The graph of $f(x) = b^x$:

Example 2: Sketch the graph of $f(x) = 2^x$ by plotting points.

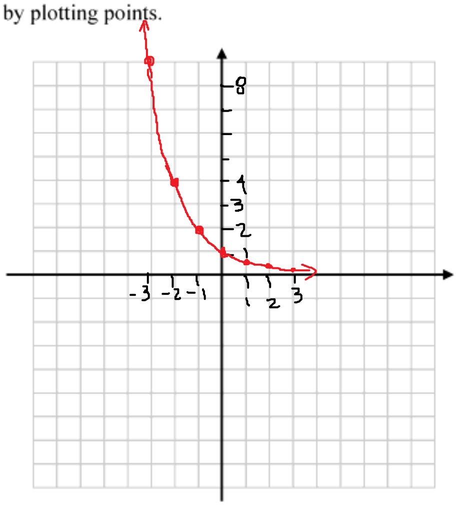
x	$f(x) = 2^x$
-2	$f(-2) = (2)^{-2} = \frac{1}{(2)^2} = \frac{1}{4}$
-1	$f(-1) = (2)^{-1} = \frac{1}{2}$
0	$f(0) = (2)^0 = 1$
1	$f(1) = (2)^1 = 2$
2	$f(2) = (2)^2 = 4$
3	$f(2) = (2)^3 = 8$

do this 5 points every time you graph an exponential



Example 3: Sketch the graph of $f(x)=\left(\frac{1}{2}\right)^x$ by plotting points.

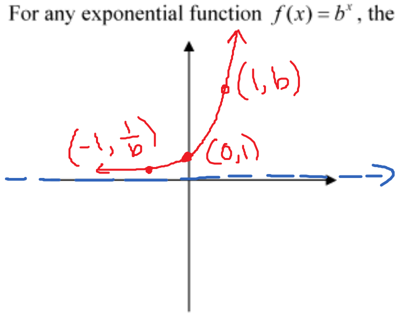
x	$f(x)=\left(\frac{1}{2}\right)^x$
-3	$\left(\frac{1}{2}\right)^{-3}=\left(\frac{2}{1}\right)^3=2^3=8$
-2	$\left(\frac{1}{2}\right)^{-2}=\left(\frac{2}{1}\right)^2=2^2=4$
-1	$\left(\frac{1}{2}\right)^{-1}=\left(\frac{2}{1}\right)^1=2$
0	$\left(\frac{1}{2}\right)^0=1$
1	$\left(\frac{1}{2}\right)^1=\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2=\frac{1^2}{2^2}=\frac{1}{4}$
3	$\left(\frac{1}{2}\right)^3=\frac{1^3}{2^3}=\frac{1}{8}$



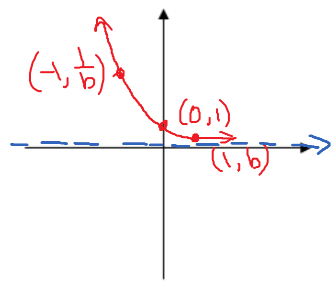
Note: $\left(\frac{1}{2}\right)^x=\left(\frac{2}{1}\right)^{-x}=2^{-x}$

So $y=\left(\frac{1}{2}\right)^x$ is the reflection over the y-axis of $y=2^x$.

For any exponential function $f(x)=b^x$, the graph looks like one of the following.



$b>1$



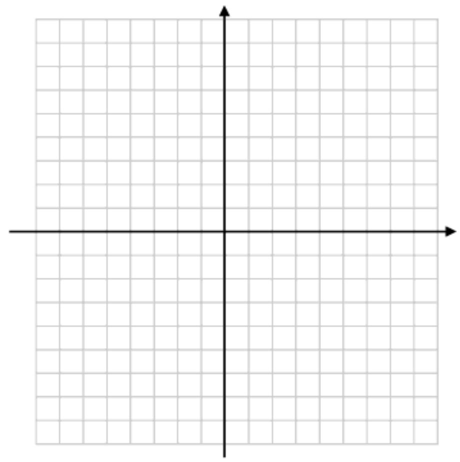
$0<b<1$

Notice:

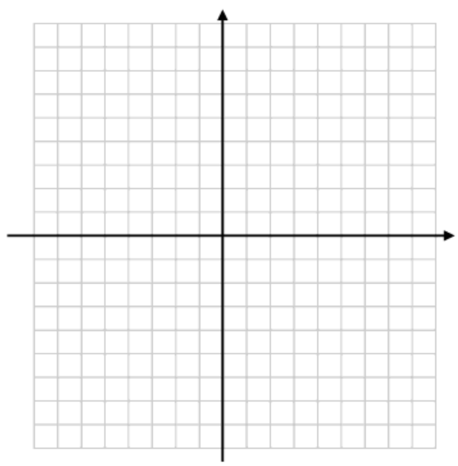
- Domain is $(-\infty,\infty)$.
- Range is $(0,\infty)$.
- Horizontal asymptote is the line $y=0$ (the x-axis).
- Always passes through the points $(0,1)$, $(1,b)$, $(-1,\frac{1}{b})$.

How do different bases affect the graph?

For $b > 1$, a larger b results in a steeper graph.



For $b < 1$, a smaller b results in a steeper graph.



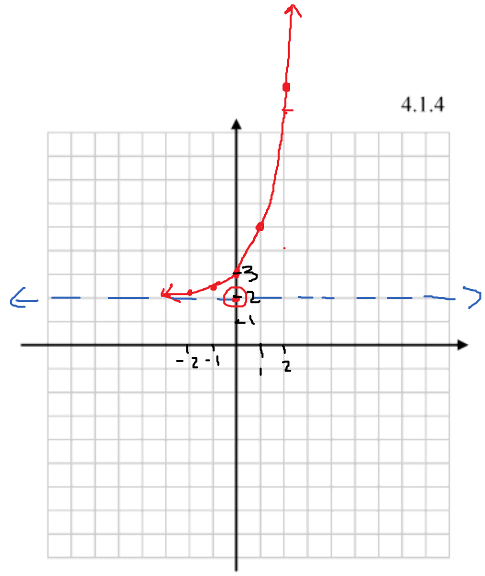
Example 4: Sketch the graph of $y = 2 + 3^x$.

$y = 3^x + 2$

Parent function: $y = 3^x$

x	3^x
-2	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

start with $y = 3^x$, then shift it up 2

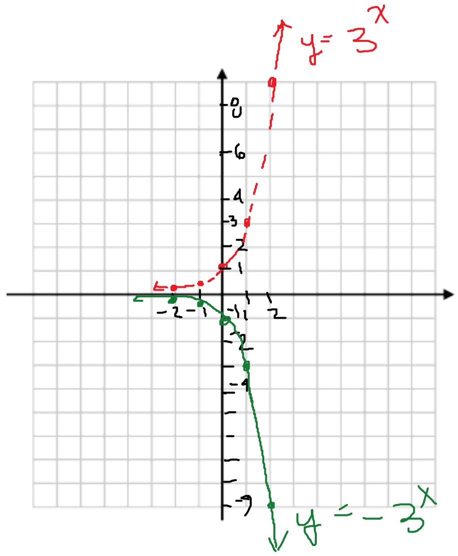


Example 5: Sketch the graph of $f(x) = -3^x$.

parent function $y = 3^x$

x	3^x
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

Graph $y = 3^x$, then reflect around x-axis



Example 6: Sketch the graph of $f(x) = 4^{x+1} - 3$.

$y = 4^{x+1} - 3$

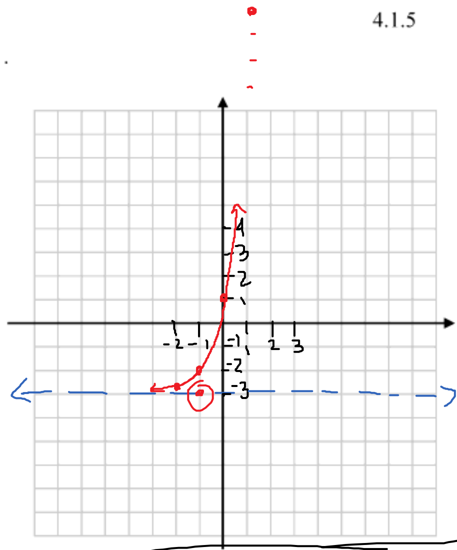
Parent function: $y = 4^x$

x	4^x
-2	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
-1	$4^{-1} = \frac{1}{4}$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$

Then shift it left 1, down 3

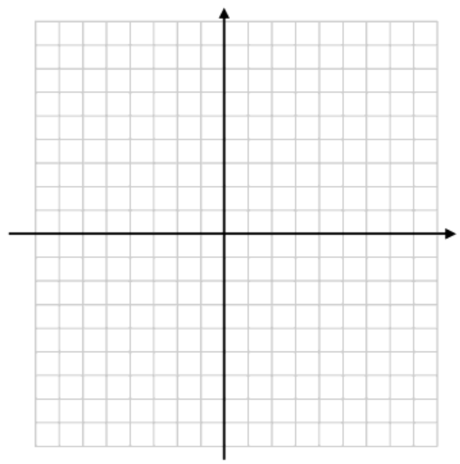
$x + 1 = 0$

$x = -1 \Rightarrow$ shift left 1

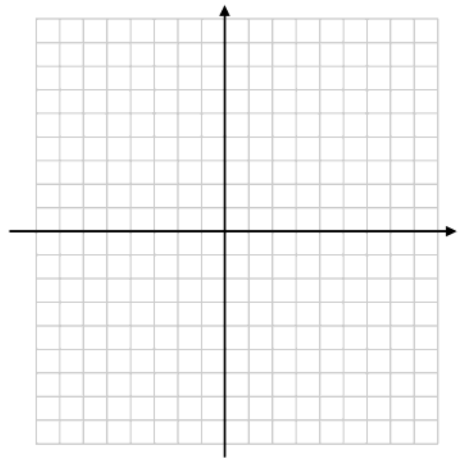


Horizontal asymptote: $y = -3$

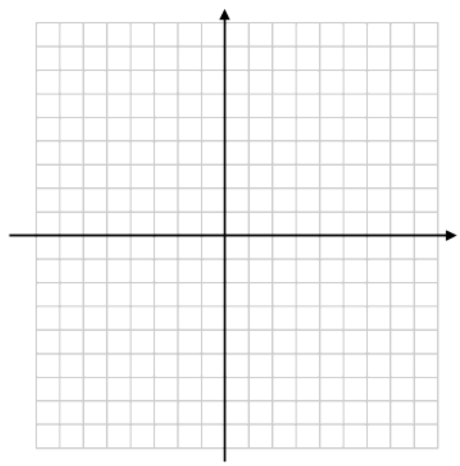
Example 7: Sketch the graph of $y = 2 \cdot 3^x$.



Example 8: Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^{x+1}$.



Example 9: Sketch the graph of $f(x) = \left(\frac{2}{3}\right)^{x}$.



The Number *e*:

We've worked with 2^x, 3^x, etc. Now we have e^x.

What is e? e is a very important number. <u>Definition</u>: e is the "limiting value" of $(1+\frac{1}{x})^x$ as x grows to infinity.

$$\boxed{e \approx 2.718281828459}$$

It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal. Unless otherwise asked, leave e and π as e and π! Do not approximate!

<u>Remember</u>: e is a number, just as 2, 3, and 17 are numbers. So it can be treated the same way.

In mathematics, it is very rare for anyone to use e as a variable.

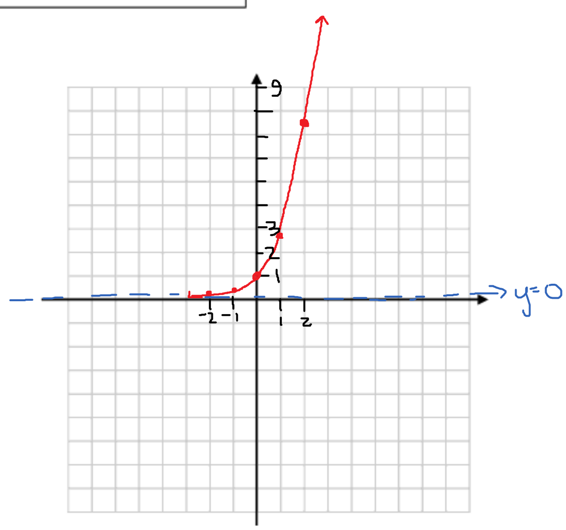
<u>Believe it or not</u>: $f(x)=e^x$ is a much "nicer" function than $f(x)=2^x$. In fact, you must change 2^x to e^{cx} (c a number), before you can do any calculus on it.

$$\boxed{\text{The } \textit{natural exponential function} \text{ is } f(x)=e^x.}$$

The graph of $f(x)=e^x$:

Since $e>1$, its graph looks like:

x	$f(x)=e^x$
-2	$e^{-2}=\frac{1}{e^2}\approx\frac{1}{9}$
-1	$e^{-1}=\frac{1}{e}\approx\frac{1}{3}$
0	$e^0=1$
1	$e^1=e\approx 2.7$
2	$e^2\approx$ ~~a little less than 9~~ 7.4



Example 10: Sketch the graph of $f(x) = e^{x+1} + 2$.