

4.1: Exponential Functions

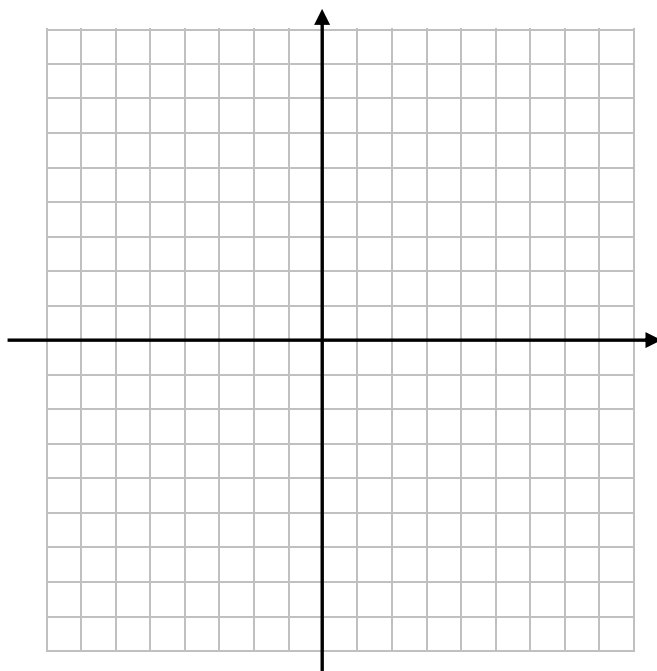
An *exponential* function takes the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$.

Example 1:

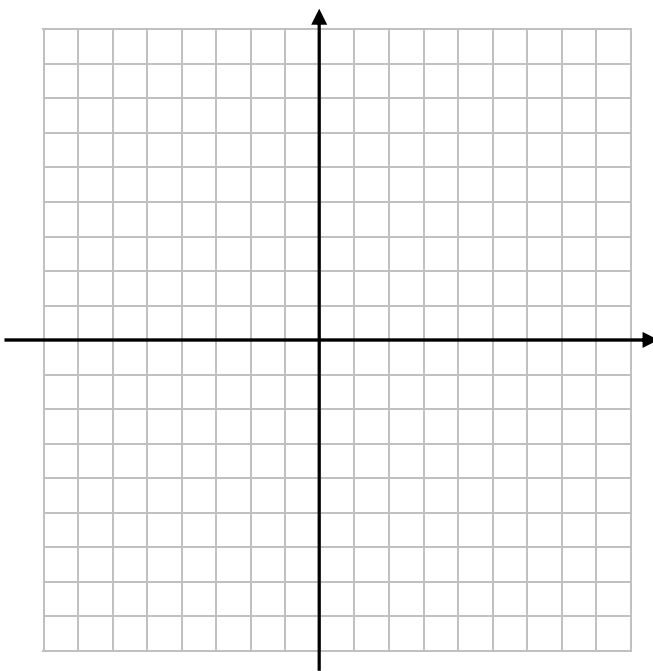
Why must we have $b > 0$ and $b \neq 1$?

The graph of $f(x) = b^x$:

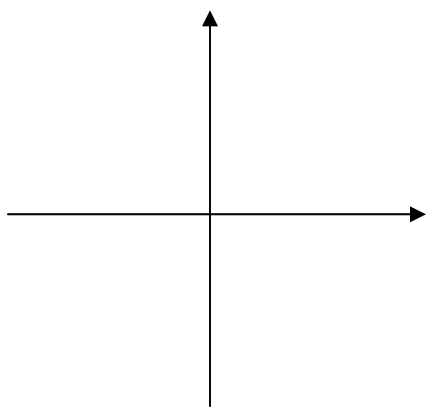
Example 2: Sketch the graph of $f(x) = 2^x$ by plotting points.



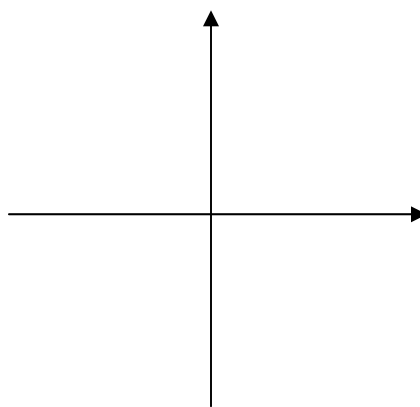
Example 3: Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ by plotting points.



For any exponential function $f(x) = b^x$, the graph looks like one of the following.



$$b > 1$$



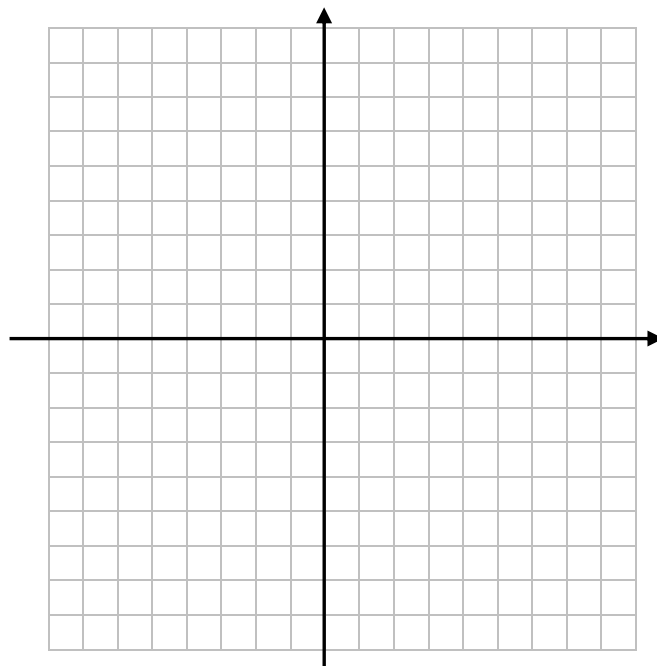
$$0 < b < 1$$

Notice:

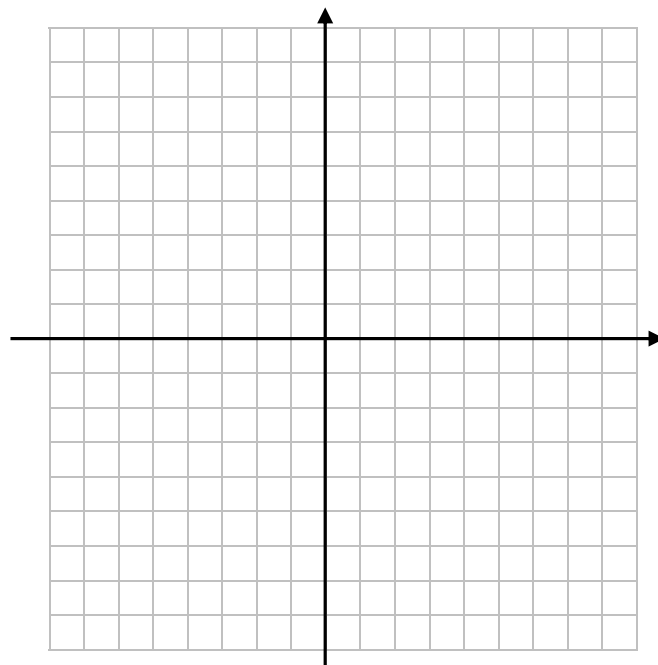
- Domain is _____.
- Range is _____.
- Horizontal asymptote is _____.
- Always passes through the points _____.

How do different bases affect the graph?

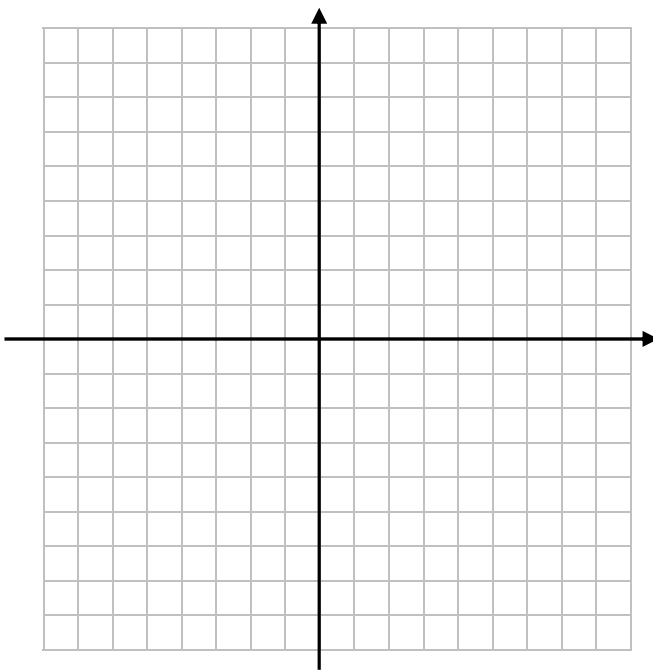
For $b > 1$, a larger b results in a steeper graph.



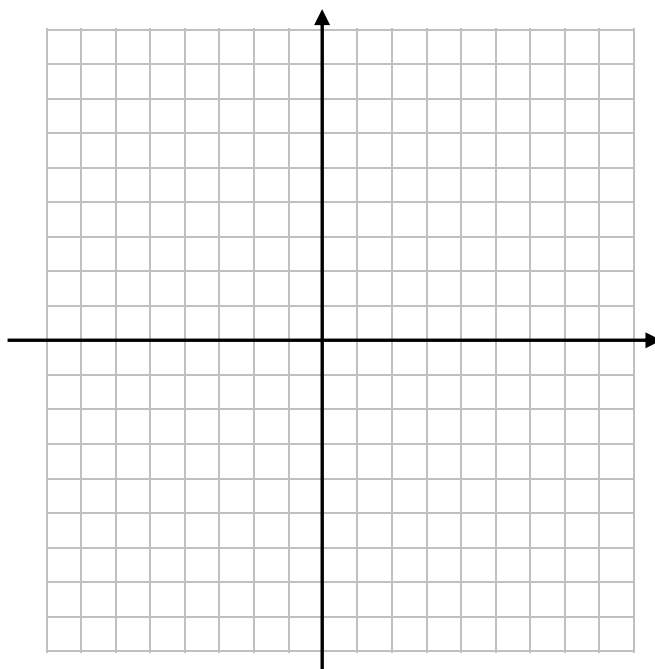
For $b < 1$, a smaller b results in a steeper graph.



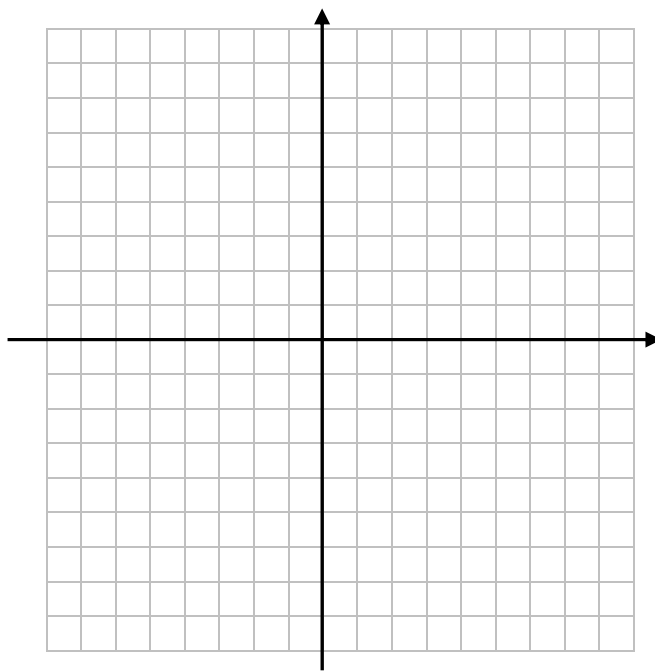
Example 4: Sketch the graph of $y = 2 + 3^x$.



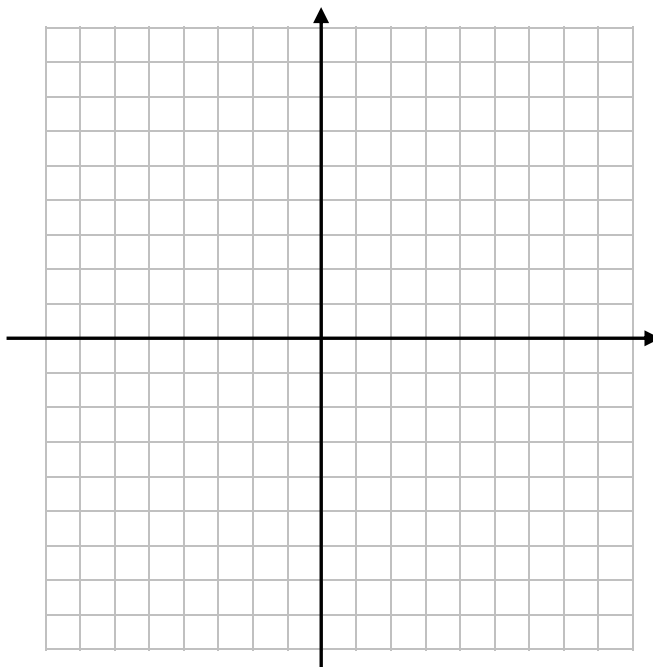
Example 5: Sketch the graph of $f(x) = -3^x$.



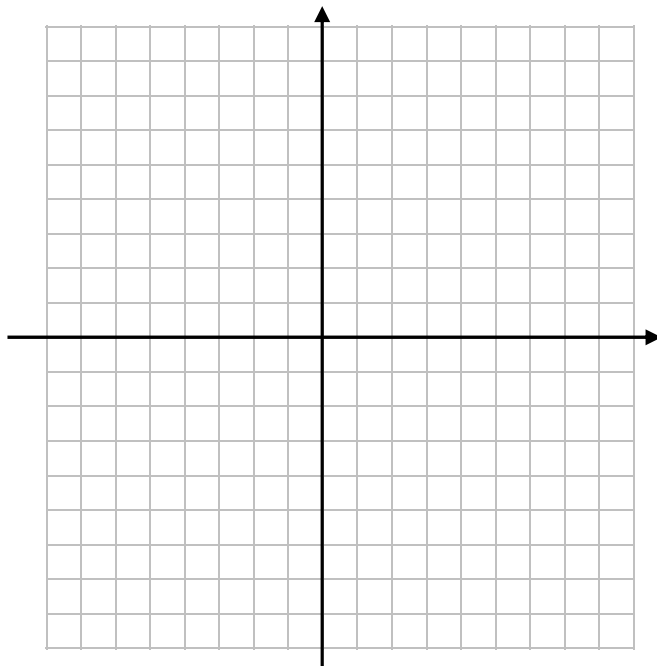
Example 6: Sketch the graph of $f(x) = 4^{x+1} - 3$.



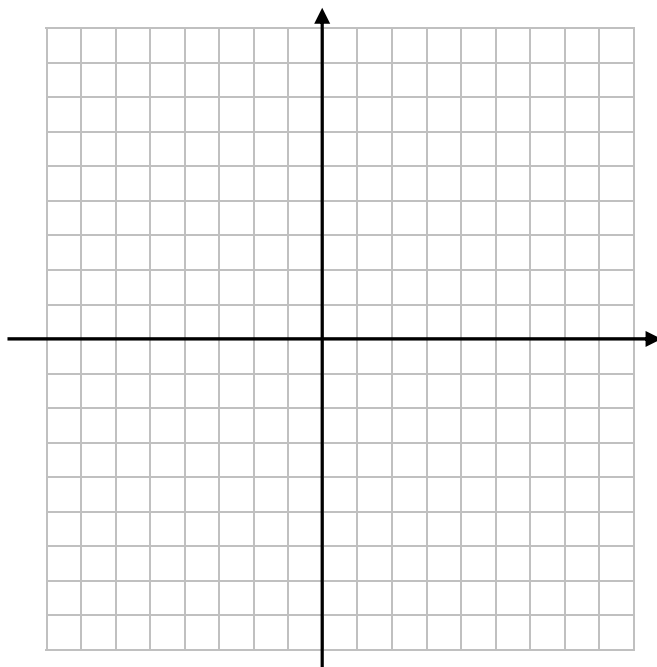
Example 7: Sketch the graph of $y = 2 \cdot 3^x$.



Example 8: Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^{x+1}$.



Example 9: Sketch the graph of $f(x) = \left(\frac{2}{3}\right)^x$.



The Number e :

We've worked with 2^x , 3^x , etc. Now we have e^x .

What is e ? e is a very important number. Definition: e is the “limiting value” of $(1 + \frac{1}{x})^x$ as x grows to infinity.

$$e \approx 2.718281828459$$

It is an irrational number, like π . This means it cannot be written as a fraction nor as a terminating or repeating decimal. Unless otherwise asked, leave e and π as e and π ! Do not approximate!

Remember: e is a number, just as 2, 3, and 17 are numbers. So it can be treated the same way.

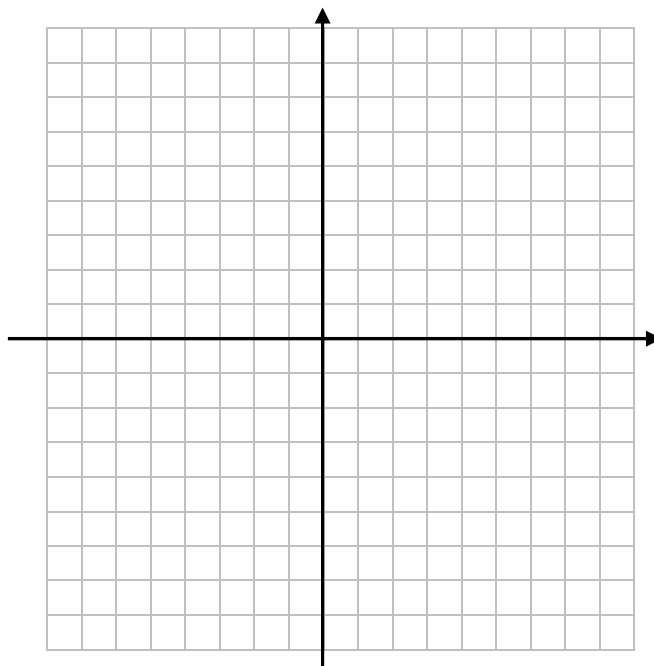
In mathematics, it is very rare for anyone to use e as a variable.

Believe it or not: $f(x) = e^x$ is a much “nicer” function than $f(x) = 2^x$. In fact, you must change 2^x to e^{cx} (c a number), before you can do any calculus on it.

The *natural exponential function* is $f(x) = e^x$.

The graph of $f(x) = e^x$:

Since $e > 1$, its graph looks like:



Example 10: Sketch the graph of $f(x) = e^{x+1} + 2$.

