1314-4-3-Notes-log-props

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4.3: Properties of Logarithms

Laws (properties, rules) of logarithms:

1. $\log_b b = 1$

   Note: $\log_b b^1 = 1$ Or. $\log_b b = 1$, $b^? = b$

2. $\log_b 1 = 0$

   $b^? = 1$

3. $\log_b(PQ) = \log_b P + \log_b Q$

4. $\log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q$

   Note: this results in $\log_b\left(\frac{1}{Q}\right) = -\log_b Q$.

5. $\log_b P^n = n\log_b P$

6. $b^{\log_b P} = P$

7. $\log_b b^P = P$

8. $\log_a x = \frac{\log_b x}{\log_b a}$ (Change of base formula)

   Note: this results in $\log_a x = \frac{\ln x}{\ln a}$.

Example 1: Simplify $\log_2 56 - \log_2 7$.

Prop. 4

$\log_2 56 - \log_2 7 = \log_2\left(\frac{56}{7}\right) = \log_2 8 = \log_2 2^3 = \boxed{3}$

Example 2: Simplify $\log_6 4 + \log_6 9$.

Prop 3

$\log_6 4 + \log_6 9 = \log_6(4 \cdot 9) = \log_6 36 = \log_6 6^2 = \boxed{2}$

Example 3: Simplify $\log 50 - \log 2 + \log 4$.

Example 4: Simplify $\log_2 8^{10}$.

Example 5: Simplify $5^{2\log_5 3}$.

$5^{2\log_5 3} = 5^{\log_5 3^2} = 3^2 = \boxed{9}$

Example 6: Simplify $e^{6\ln 2}$.

$e^{6\ln 2} = e^{\ln 2^6} = 2^6 = \boxed{64}$

Example 7: Simplify $\log_6\left(\frac{\sqrt[3]{x}}{36}\right)$.

Errors to avoid:

$$\log_a(x+y) \neq \log_a x + \log_a y$$
$$\frac{\log_a x}{\log_a y} \neq \log_a x - \log_a y$$
$$(\log_a x)^3 \neq 3\log_a x$$

Putting the properties together:

Example 8: Use the properties of logarithms to expand the expression as much as possible.

$\log_3(x(x+4))$

$\log_3(x(x+4)) = \boxed{\log_3(x) + \log_3(x+4)} \leftarrow$ Prop 3

Example 9: Use the properties of logarithms to expand the expression as much as possible.

$\ln\left(\frac{x+2}{e}\right) = \boxed{\ln(x+2) - \ln(e)} \leftarrow$ Prop 4

Example 10: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\left(\frac{ab^3}{c^2d}\right) = \ln\left(\frac{ab^3}{c^2d}\right) = \ln(ab^3) - \ln(c^2d) \longleftarrow \text{Prop } 4$$

$$= (\ln a + \ln b^3) - (\ln c^2 + \ln d)$$

$$= \ln a + \ln b^3 - \ln c^2 - \ln d$$

$$= \boxed{\ln a + 3\ln b - 2\ln c - \ln d}$$

Example 11: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\left[\frac{x+5}{x^2-4}\right] = \ln(x+5) - \ln(x^2-4)$$

$$= \ln(x+5) - \ln((x+2)(x-2))$$

$$= \ln(x+5) - [\ln(x+2) + \ln(x-2)]$$

$$= \boxed{\ln(x+5) - \ln(x+2) - \ln(x-2)}$$

Example 12: Use the properties of logarithms to expand the expression as much as possible.

$$\log_{10}\left[\frac{x+1}{x^2(x-3)\sqrt{x+7}}\right] = \log_{10}\left[\frac{x+1}{x^2(x-3)(x+7)^{1/2}}\right]$$

$$= \log_{10}(x+1) - \log_{10}(x^2) - \log_{10}(x-3) - \log_{10}(x+7)^{1/2}$$

$$= \boxed{\log_{10}(x+1) - 2\log_{10}x - \log_{10}(x-3) - \frac{1}{2}\log_{10}(x+7)}$$

Example 13: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\sqrt{\frac{(x-3)^2(x+9)^4}{x^6(x-10)}}$$

Example 14: Rewrite as a single logarithm with a coefficient of 1.

$\log_3 x + \log_3 2 \quad = \boxed{\log_3 (2x)}$

Example 15: Rewrite as a single logarithm with a coefficient of 1.

$\log_a b - c\log_a d + r\log_a s$

$= \log_a b - \log_a d^c + \log_a s^r \quad = (\log_a b + \log_a s^r) - \log_a d^c$

$= \log_a (bs^r) - \log_a d^c$

$= \boxed{\log_a \left[\frac{b s^r}{d^c}\right]}$

Rewrite as a single logarithm with a coefficient of 1.

$2\ln x - 5\ln(x+1) - \frac{1}{2}\ln(x-3)$

$= \ln x^2 - \ln(x+1)^5 - \ln(x-3)^{1/2}$

$= \ln x^2 - \left[\ln(x+1)^5 + \ln(x-3)^{1/2}\right]$

$= \ln x^2 - \ln\left((x+1)^5(x-3)^{1/2}\right) = \boxed{\ln\left[\frac{x^2}{(x+1)^5\sqrt{x-3}}\right]} = \boxed{\ln\left[\frac{x^2}{(x+1)^5(x-3)^{1/2}}\right]}$

Rewrite as a single logarithm with a coefficient of 1.

$$\frac{1}{2}[3\log_5(x+2) - 2\log_5(x-1) + \log_5 x - 4\log_5(x-7)]$$

Example 16: Rewrite $\ln 60$ so that it contains only logs that are being applied to prime numbers.

$\ln(60) = \ln(2^2 \cdot 5 \cdot 3) = \ln 2^2 + \ln 5 + \ln 3$

$= \boxed{2\ln 2 + \ln 5 + \ln 3}$

$\ln(60) = \ln(2 \cdot 2 \cdot 5 \cdot 3) = \ln 2 + \ln 2 + \ln 5 + \ln 3$

$= \boxed{2\ln 2 + \ln 5 + \ln 3}$

60

20·3

4·5·3

2·2·5·3

Example 17: Rewrite $\log 72$ so that it contains only logs that are being applied to prime numbers.

Note: Suppose $\log_2 17 = x$

$2^x = 17$

$\log_{10} 2^x = \log_{10} 17$

$x \log_{10} 2 = \log_{10} 17 \Rightarrow x = \frac{\log_{10} 17}{\log_{10} 2}$.

Example 18: Use a calculator to approximate $\log_2 17$ to the nearest hundredth.

Change-of-base formula:

$\log_a x = \frac{\log_b x}{\log_b a}$

In this example, $a = 2$, $b = e$

$\log_2 17 = \frac{\ln 17}{\ln 2} \approx 4.087462841$

$\approx \boxed{4.0875}$

OR $\log_2 17 = \frac{\log_{10} 17}{\log_{10} 2} \approx \boxed{4.0875}$

On test: If asked to find a logarithm of a base other than e or 10 on your calculator, use the change-of-base formula.

Example 19: Use a calculator to evaluate $\log_7 12$ to the nearest thousandth.

$$\log_7 12 = \frac{\ln 12}{\ln 7} \approx \boxed{1.277}$$

Notice: $\frac{\ln 12}{\ln 7} \neq \ln 12 - \ln 7$

$\ln\left(\frac{12}{7}\right) = \ln 12 - \ln 7$

$\ln\left(\frac{12}{7}\right) \approx 0.5389$

About calculators and books:

- Most calculators and some books (including ours) use
  - ln for natural log (base e).
  - log for $\log_{10}$.
- Some math books (usually very advanced ones or those from other countries)
  - use log for natural log.
  - That's all. Logs of other bases are useless.
- Some books (this is my preference)
  - use ln for natural log.
  - use $\log_{10}$ for $\log_{10}$.
  - never write log by itself (too confusing!!)