4.3: Properties of Logarithms

Laws (properties, rules) of logarithms:

1. $\log_b b = 1$

2. $\log_b 1 = 0$

3. $\log_b (PQ) = \log_b P + \log_b Q$

4. $\log_b \left(\frac{P}{Q} \right) = \log_b P - \log_b Q$

   Note: this results in $\log_b \left(\frac{1}{Q} \right) = -\log_b Q$.

5. $\log_b P^n = n \log_b P$

6. $b^{\log_b P} = P$

7. $\log_b b^P = P$

8. $\log_a x = \frac{\log_b x}{\log_b a}$ (Change of base formula)

   Note: this results in $\log_a x = \frac{\ln x}{\ln a}$.

Example 1: Simplify $\log_2 56 - \log_2 7$.

Example 2: Simplify $\log_6 4 + \log_6 9$.

Example 3: Simplify $\log 50 - \log 2 + \log 4$.

Example 4: Simplify $\log_2 8^{10}$.

Example 5: Simplify $5^{2\log_5 3}$.

Example 6: Simplify $e^{6\ln 2}$.

Example 7: Simplify $\log_6\left(\frac{\sqrt[3]{x}}{36}\right)$.

Errors to avoid:

$$\log_a(x+y) \neq \log_a x + \log_a y$$

$$\frac{\log_a x}{\log_a y} \neq \log_a x - \log_a y$$

$$(\log_a x)^3 \neq 3\log_a x$$

Putting the properties together:

Example 8: Use the properties of logarithms to expand the expression as much as possible.

$\log_3(x(x+4))$

Example 9: Use the properties of logarithms to expand the expression as much as possible.

$\ln\left(\frac{x+2}{e}\right)$

Example 10: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\left(\frac{ab^3}{c^2d}\right)$$

Example 11: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\left[\frac{x+5}{x^2-4}\right]$$

Example 12: Use the properties of logarithms to expand the expression as much as possible.

$$\log_{10}\left[\frac{x+1}{x^2(x-3)\sqrt{x+7}}\right]$$

Example 13: Use the properties of logarithms to expand the expression as much as possible.

$$\ln\sqrt{\frac{(x-3)^2(x+9)^4}{x^6(x-10)}}$$

Example 14: Rewrite as a single logarithm with a coefficient of 1.

$\log_3 x + \log_3 2$

Example 15: Rewrite as a single logarithm with a coefficient of 1.

$\log_a b - c\log_a d + r\log_a s$

Rewrite as a single logarithm with a coefficient of 1.

$2\ln x - 5\ln(x+1) - \frac{1}{2}\ln(x-3)$

Rewrite as a single logarithm with a coefficient of 1.

$$\frac{1}{2}[3\log_5(x+2) - 2\log_5(x-1) + \log_5 x - 4\log_5(x-7)]$$

Example 16: Rewrite $\ln 60$ so that it contains only logs that are being applied to prime numbers.

Example 17: Rewrite $\log 72$ so that it contains only logs that are being applied to prime numbers.

Example 18: Use a calculator to approximate $\log_2 17$ to the nearest hundredth.

Example 19: Use a calculator to evaluate $\log_7 12$ to the nearest thousandth.

About calculators and books:

- Most calculators and some books (including ours) use
  - ln for natural log (base *e*).
  - log for $\log_{10}$.
- Some math books (usually very advanced ones or those from other countries)
  - use log for natural log.
  - That's all. Logs of other bases are useless.
- Some books (this is my preference)
  - use ln for natural log.
  - use $\log_{10}$ for $\log_{10}$.
  - never write log by itself (too confusing!!)