"Solve" means to find all the solutions.

HW Qs

1.4 #105 ~~Solve.~~ Find a solution for the equation.

$$\tan(3\theta - 4^\circ) = \frac{1}{\cot(5\theta - 8^\circ)}$$

$$\tan(3\theta - 4^\circ) = \tan(5\theta - 8^\circ)$$

To find a solution: Set $3\theta - 4^\circ = 5\theta - 8^\circ$ and solve for θ.

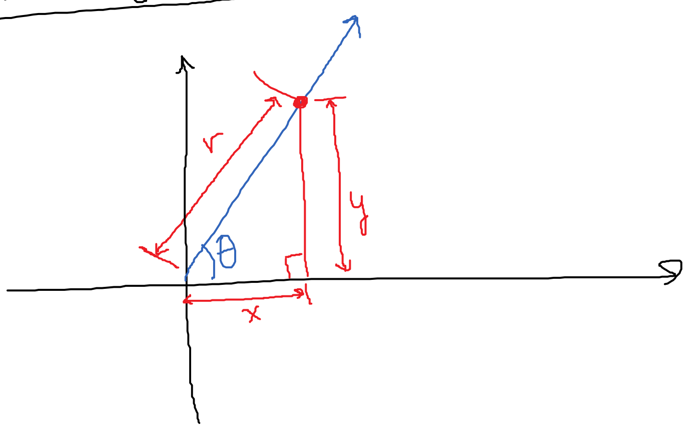
Use this identity:

$$\tan\theta = \frac{1}{\cot\theta}$$

also $\cot\theta = \frac{1}{\tan\theta}$

($\tan\theta$, $\cot\theta$ are reciprocals of each other)

2.1: Trigonometric Functions of acute Angles



$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$\tan\theta = \frac{y}{x}$

Recall: θ is an acute angle if

$$0 < \theta < 90^\circ$$

(So 1st quadrant)

In QI, all of x, y, r are positive.

Right-triangle-Based Trig definitions (for θ an acute angle in a triangle)

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opp}}{\text{adj}}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cot\theta = \frac{\text{adj}}{\text{opp}}$

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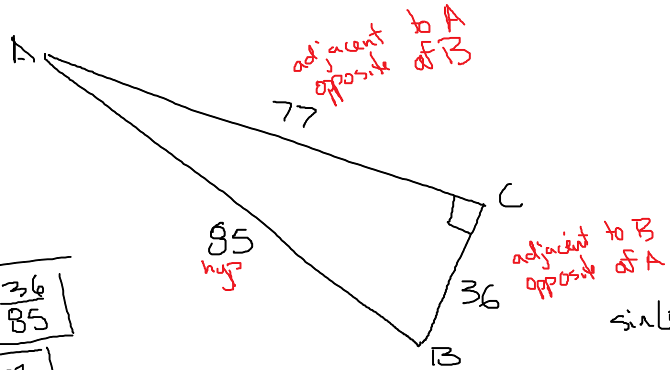
$\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ $\quad$ $\text{Cos} = \frac{\text{Adj}}{\text{Hyp}}$ $\quad$ $\text{Tan}\,\theta = \frac{\text{Opp}}{\text{Adj}}$

Ex:



Check:
$36^2 + 77^2 \stackrel{?}{=} 85^2$
$7225 = 7225$ ✓.

$\sin(A) = \frac{opp}{hyp} = \boxed{\frac{36}{85}}$

$\cos(A) = \frac{adj}{hyp} = \boxed{\frac{77}{85}}$

$\tan(A) = \frac{opp}{adj} = \boxed{\frac{36}{77}}$

$\sin(B) = \frac{opp}{hyp} = \boxed{\frac{77}{85}}$

$\cos(B) = \frac{adj}{hyp} = \boxed{\frac{36}{85}}$

$\tan(B) = \frac{opp}{adj} = \boxed{\frac{77}{36}}$

$\cot(B) = \boxed{\frac{36}{77}}$

Could we find the trig function of C?
Yes, but not with the right-triangle definitions.
These right-triangle definitions (opp, adj, hyp) only work for acute angles (between 0° and 90°)

Notice: $\cos(A) = \sin(B)$
Also $\sin(A) = \cos(B)$
$\tan(A) = \cot(B)$

this is because A and B are complements.
(complementary angles: add up to 90°)

Cofunction Identities

$$\sin\theta = \cos(90^\circ - \theta)$$
$$\cos\theta = \sin(90^\circ - \theta)$$
$$\tan\theta = \cot(90^\circ - \theta)$$

Note: θ and $90^\circ - \theta$ are complements.

also:
$$\sec\theta = \csc(90^\circ - \theta)$$
$$\csc\theta = \sec(90^\circ - \theta)$$
$$\cot\theta = \tan(90^\circ - \theta)$$

co-sine is sine of the complement

co-secant is the secant of the complement

co-tangent is the tangent of the complement

Write each function in terms of its cofunction.

Ex: $\sin(9^\circ) = \cos(81^\circ)$

$\cot(76^\circ) = \tan(14^\circ)$

$\sec(33^\circ) = \csc(57^\circ)$

$$\begin{array}{r} 90^\circ \\ -\ 33^\circ \\ \hline 57^\circ \end{array}$$

Trig Functions of Some Special Angles $(30^\circ, 60^\circ, 45^\circ)$

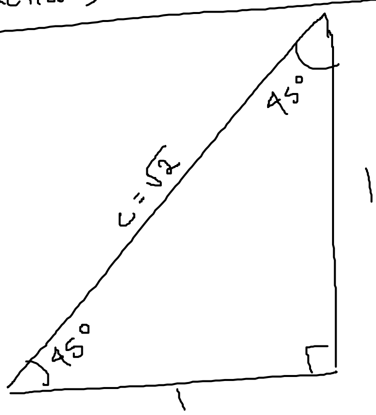
$a^2+b^2=c^2$

$1^2+1^2=c^2$

$2=c^2$

$c=\pm\sqrt{2}$

$c=+\sqrt{2}$

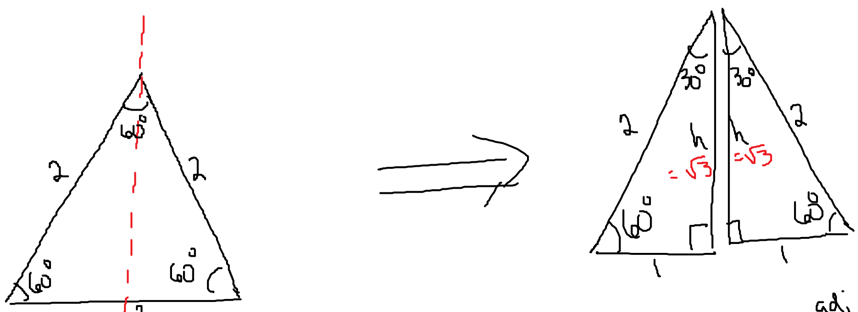


45°-45°-90° right triangle

$\sin(45^\circ) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\cos(45^\circ) = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

$\tan(45^\circ) = \frac{opp}{adj} = \frac{1}{1} = \boxed{1}$



equilateral triangles

30°- 60°- 90°

Find h: $a^2+b^2=c^2$ Pythagorean Thm

$1^2+h^2=2^2$

$1+h^2=4$

$h^2=3$

$h=\pm\sqrt{3}$ choose $h=+\sqrt{3}$

$\cos(30^\circ) = \frac{adj}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$

$\sin(30^\circ) = \frac{opp}{hyp} = \boxed{\frac{1}{2}}$

$\tan(30^\circ) = \frac{opp}{adj} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

$\cos(60^\circ) = \frac{adj}{hyp} = \boxed{\frac{1}{2}}$

$\sin(60^\circ) = \frac{opp}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$

$\tan(60^\circ) = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$