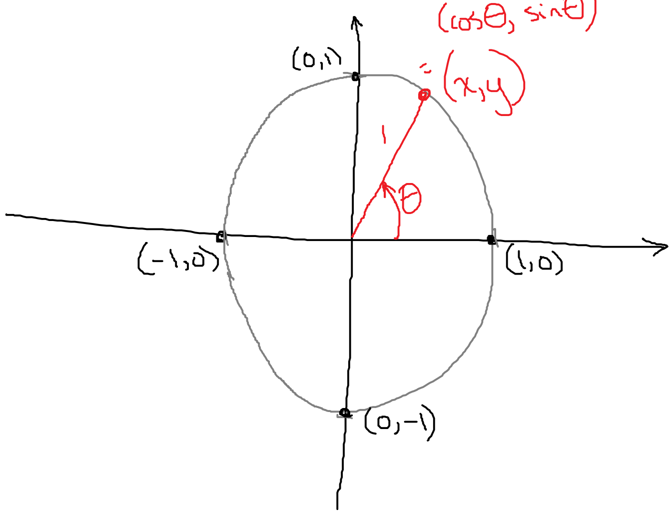
2.2: Trig Functions of Non-Acute Angles

Unit Circle: $x^2 + y^2 = 1$. Circle of radius 1.



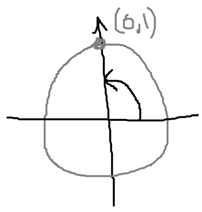
On unit circle

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

Ex: Consider 90°



$\cos(90^\circ) = x = \boxed{0}$

$\sin(90^\circ) = y = \boxed{1}$

$\tan(90^\circ) = \frac{y}{x} \Rightarrow \cancel{\frac{1}{0}}$ $\boxed{\text{undefined}}$

$\sec(90^\circ) = \cancel{\frac{1}{0}}$ $\boxed{\text{undefined}}$

$\csc(90^\circ) = \frac{1}{1} = \boxed{1}$

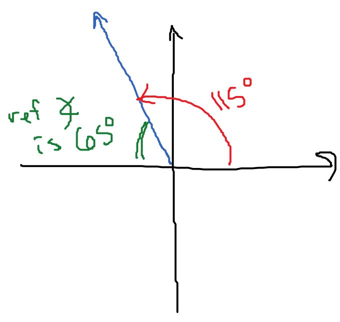
$\cot(90^\circ) = \frac{x}{y} = \frac{0}{1} = \boxed{0}$

Reference Angles

A <u>reference angle</u> is the positive acute angle formed by the terminal side and the x-axis.

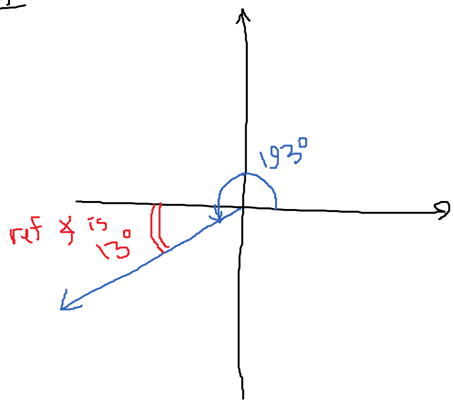
("how big is the gap between the angle and the x-axis?")

Ex: 115° what is ref angle for 115°?



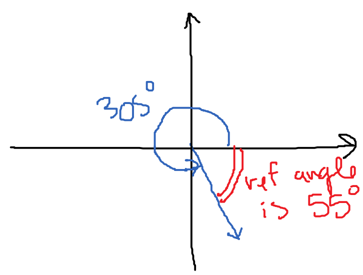
Ref angle is $180^\circ - 115^\circ = \boxed{65^\circ}$

Ex: what is the reference angle for 193°?



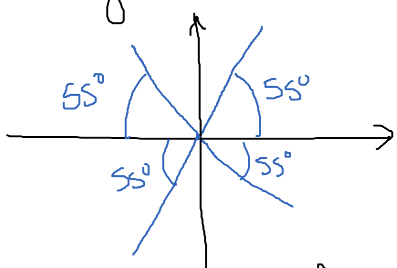
Ref. angle is $193° - 180° = \boxed{13°}$

Ex: what is the reference angle for 305°?



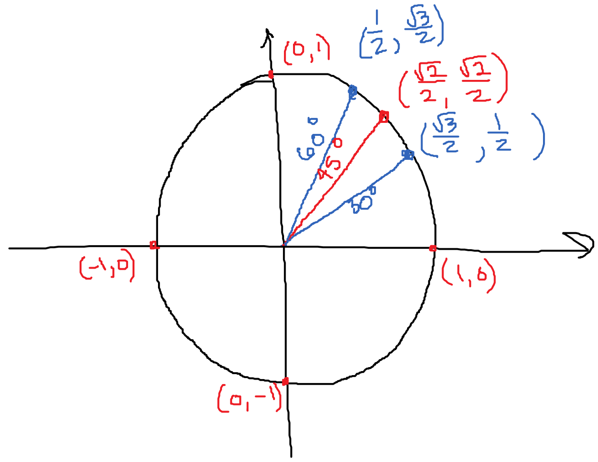
Ref. angle is $360° - 305° = \boxed{55°}$

Ref ∡ of 55° in all 4 quadrants:



Note: The quadrantal angles (lie on an axis) do not have a reference angle

We use reference angles to figure out trig functions for multiples of $30^\circ, 60^\circ, 45^\circ$



unit circle: $x^2 + y^2 = 1$

Radius $= 1$

On unit circle,

$\cos\theta = x$

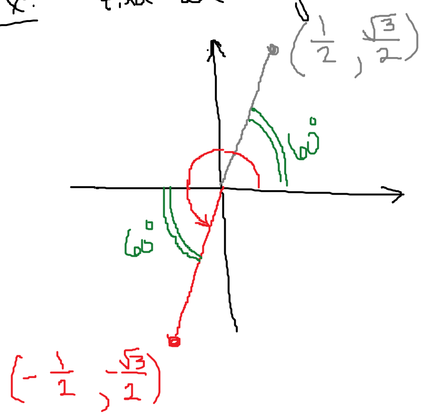
$\sin\theta = y$

$\tan\theta = \frac{y}{x}$

so $\cos 30^\circ = x = \frac{\sqrt{3}}{2}$

$\sin 30^\circ = y = \frac{1}{2}$

Ex: Find all trig functions for 240°



$\cos(240^\circ) = \boxed{-\frac{1}{2}}$

$\sin(240^\circ) = \boxed{-\frac{\sqrt{3}}{2}}$

$\tan(240^\circ) = \frac{y}{x} = \frac{-\sqrt{3}/2}{-1/2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

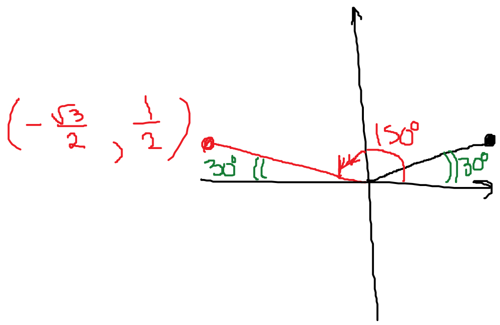
Note: $\frac{A/B}{C/B} = \frac{A}{B} \cdot \frac{B}{C} = \frac{A}{C}$

$\sec 240^\circ = \boxed{-2}$

$\csc 240^\circ = -\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

$\cot 240^\circ = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Ex: Find $\sin(150^\circ)$, $\cos(150^\circ)$, $\tan(150^\circ)$



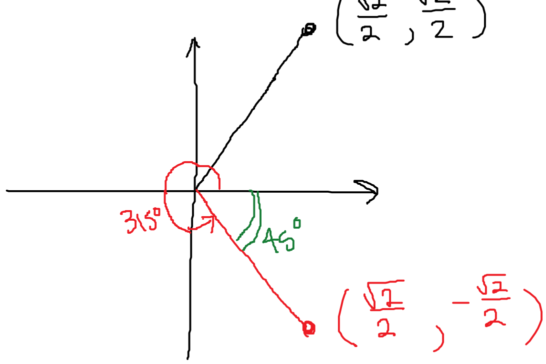
$\cos(150^\circ) = x = \boxed{-\frac{\sqrt{3}}{2}}$

$\sin(150^\circ) = y = \boxed{\frac{1}{2}}$

$\tan(150^\circ) = \frac{y}{x} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$

$= \boxed{-\frac{\sqrt{3}}{3}}$

Ex: Find $\sec 315^\circ$ and $\tan 315^\circ$



$\cos(315^\circ) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\sec(315^\circ) = \boxed{\sqrt{2}}$

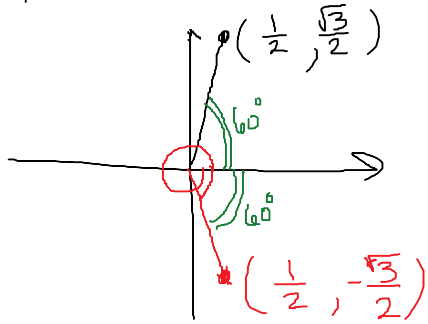
or $\sec(315^\circ) = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$

$\tan(315^\circ) = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2}$

$= -\frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{\sqrt{2}}} = \boxed{-1}$

Ex: Find $\sin(-420^\circ)$



$$\begin{array}{r} 420^\circ \\ -\ 360^\circ \\ \hline 60^\circ \end{array}$$

-60° puts you in the same place as -420°

$\sin(-420^\circ) = \boxed{-\frac{\sqrt{3}}{2}}$

Ex. $\cos 1770^\circ$

Remove the whole revolutions.

$\frac{1770^\circ}{360^\circ} \approx 4.917 \Rightarrow$ so there are 4 whole revolutions.

$1770^\circ - 4(360^\circ) = 330^\circ$