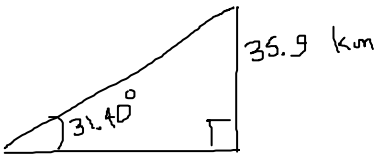
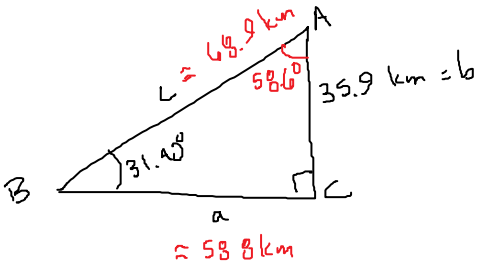
# 2.4 and 2.5: Applications and Solutions of Right Triangles

To solve a triangle means to find all the missing sides and angles.

**Ex:** Solve the Triangle.




$\approx 58.8$ km

$\sin 31.40^\circ = \frac{35.9\text{ km}}{c}$

$\sin(31.40^\circ) \cdot c = 35.9\text{ km}$

$c = \frac{35.9\text{ km}}{\sin(31.40^\circ)}$

$c \approx 68.9$ km

Missing angle is $90^\circ - 31.40^\circ = \boxed{58.60^\circ}$

or $A + B + C = 180^\circ$

$A + 31.40^\circ + 90^\circ = 180^\circ$

$A = 180^\circ - 90^\circ - 31.40^\circ$

$\boxed{A = 58.60^\circ}$

$\tan 31.40^\circ = \frac{\text{opp}}{\text{adj}} = \frac{35.9\text{ km}}{a}$

$0.610403 \approx \frac{35.9\text{ km}}{a}$

$0.610403\,a \approx 35.9\text{ km}$

$a \approx \frac{35.9\text{ km}}{0.610403}$

Use 35.9/Answer (use Previous Answer button)

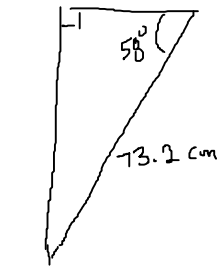
$a \approx 58.8$ km

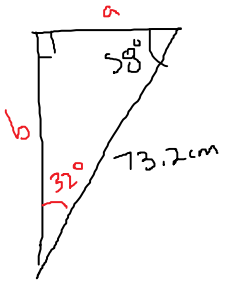
$A = 58.60^\circ$

$a \approx 58.8$ km

$c \approx 68.9$ km

Ex:




$90° - 58° = 32°$

$\sin(58°) = \frac{b}{73.2\,cm}$ $\Rightarrow \sin = \frac{opp}{hyp}$

$73.2\,cm(\sin(58°)) = b$

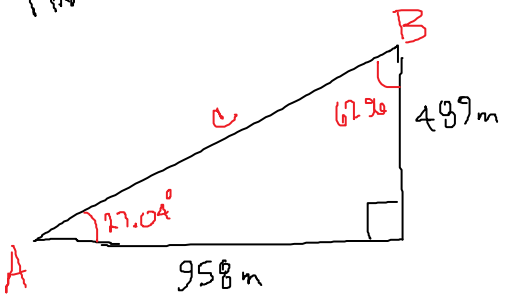
$62.1\,cm \approx b$

$\cos(58°) = \frac{adj}{hyp} = \frac{a}{73.2\,cm}$

$73.2\,cm \cdot \cos(58°) = a$

$a \approx 38.8\,cm$

Ex. Suppose a right triangle has legs of length 958 m and 489 m. Find all the other sides and angles.




$\tan(A) = \frac{489m}{958m}$

$\tan(A) \approx 0.510438$

$A \approx \tan^{-1}(0.510438) \approx 27.04°$

$B = 90° - 27.04° = 62.96°$

$\cos B = \frac{adj}{hyp} = \frac{489m}{c}$

$\cos(62.95849°) = \frac{489m}{c}$

$c \cdot \cos(62.95849°) = 489m$

$c = \frac{489m}{\cos(62.958°)} \approx \frac{489m}{0.4546} \approx 1075.6m$

$C \approx 1076$

$A \approx 27.04°$

$B \approx 69.96°$

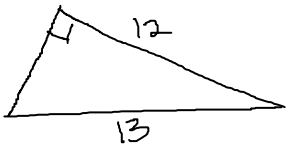
$C = 90°$

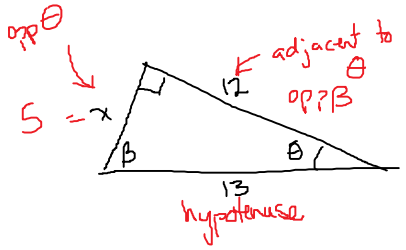
For positive values in the range of sin, cos, tan, the inverse trig functions $\cos^{-1}$, $\sin^{-1}$, $\tan^{-1}$ give you the acute angle $(0 < \theta < 90^\circ)$ that has that trig function value.

Note: $\sin^{-1}(1.3)$ is not defined.
Can you have $\sin\theta = 1.3$? No!
$\sin\theta = 1.3$ is impossible!

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ or on a circle, $\sin\theta = \frac{y}{r}$
numerator can't be larger than denominator.
$-1 \le \sin\theta \le 1$
$-1 \le \cos\theta \le 1$

Ex Find all the missing sides and angles.
(Solve the triangle).




Pythagorean Theorem:
$x^2 + 12^2 = 13^2$
$x^2 + 144 = 169$
$-144 \quad -144$
$x^2 = 25$
$x = \pm\sqrt{25} = \pm 5 \Rightarrow$ choose positive
$x = 5$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$

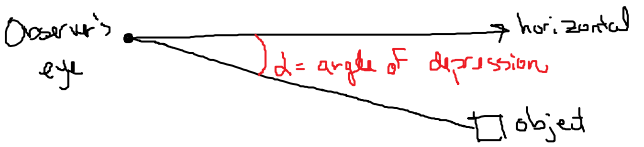
$\theta = \sin^{-1}\left(\frac{5}{13}\right) \approx 22.6199^\circ$

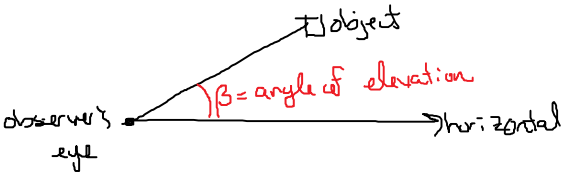
$\theta \approx 22.6^\circ$

$\beta \approx 90^\circ - 22.6^\circ = 67.4^\circ$

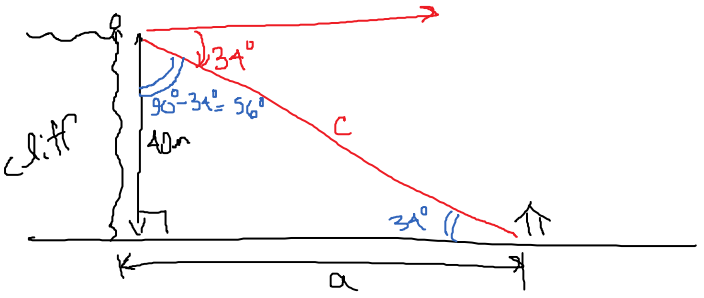
$x = 5$
$\theta \approx 22.6^\circ$
$\beta \approx 67.4^\circ$

# Angles of Elevation and Depression




**Ex:** From the top of a vertical cliff 40m high, the angle of depression to a cabin level with the base of the cliff is 34°. How far is the cabin from the base of the cliff?




$\cos(34^\circ) = \frac{adj}{hyp} = \frac{a}{c}$ don't know either of these!

Use tangent instead.

$\tan(34^\circ) = \frac{opp}{adj} = \frac{40m}{a}$

$a\tan(34^\circ) = 40m$

$a = \frac{40m}{\tan(34^\circ)}$

$a \approx 59.3\,m \Rightarrow a \approx 59m$

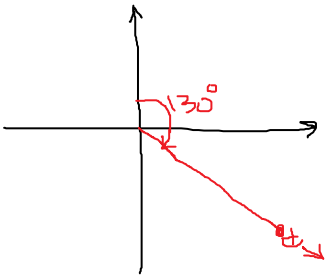
The cabin is about 59 m from the cliff.

**Bearing:** Used in navigation.

2 methods of describing bearing:

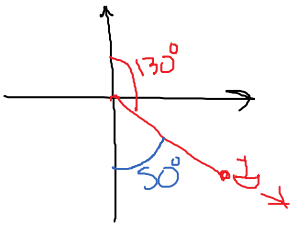
Method 1) $0° <$ some angle $< 360°$. Angle has initial side due North, rotating clockwise to the terminal side.

Ex: Sketch a bearing of $130°$.




Method 2) Start with either N (North) or S (South), whichever is closest. Then rotate for an acute angle either W (west) or East (E).

Ex: (same as above)




This boat has bearing

S 50° E

read it: "50° east of south"