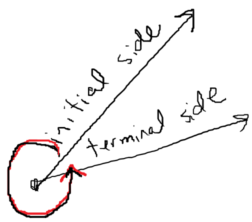
1.1: Angles

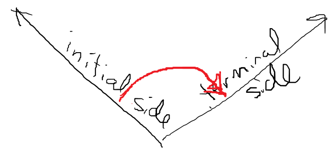
.1.1.1

Angle: 2 rays with a common endpoint (vertex).

Positive angle: (counterclockwise rotation)

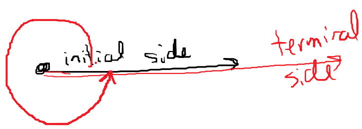
Negative Angle: (clockwise rotation)



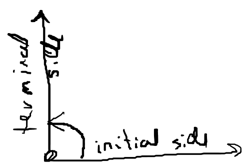


Degree Measure: The most common unit of measuring angles is the **degree**.

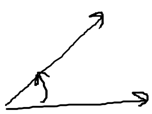
One complete rotation is 360 degrees, written 360°.



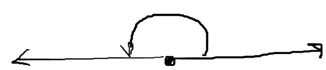
360° angle



90° angle (square corner)
(looks like capital L)
(right angle)



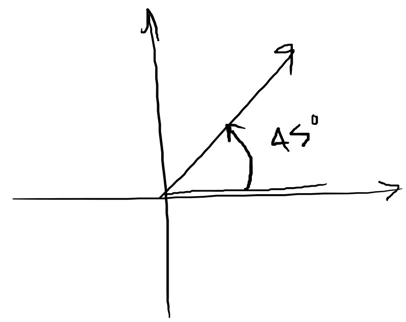
45° angle



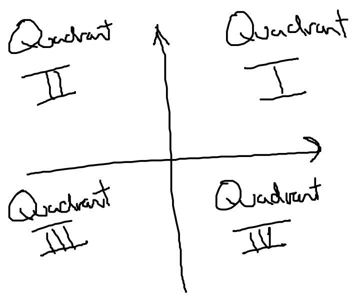
180° angle (straight angle)

0° angle (no rotation)

Standard Position: An angle is in standard position when its vertex is at the origin and its initial side lies on the positive part of the x-axis.



Quadrants:

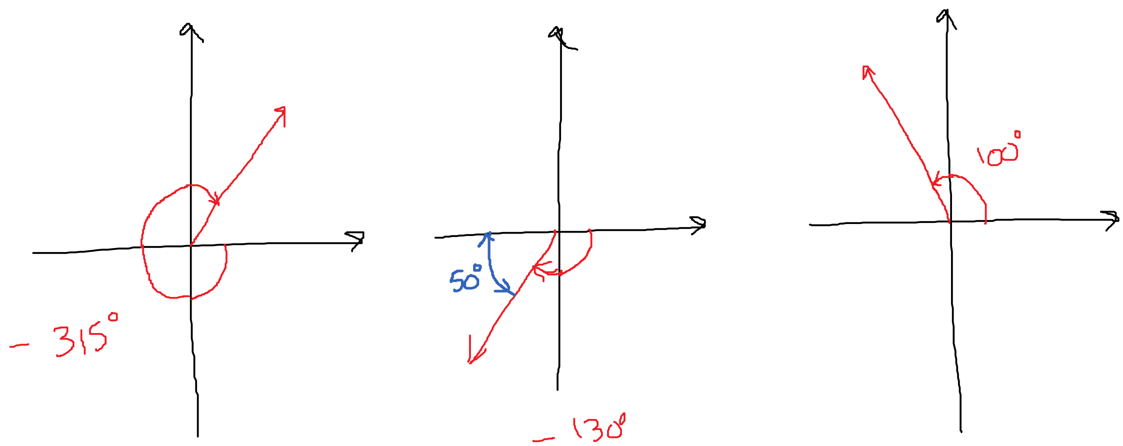


If we say an angle is in a particular quadrant, then we mean that it is in standard position with its terminal side in that quadrant.

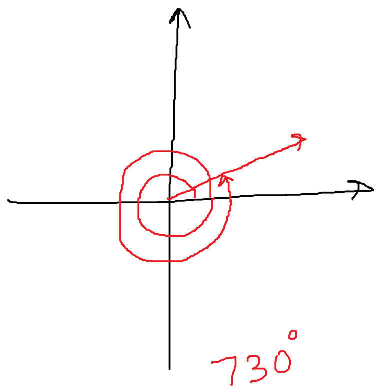
Ex. sketch these angles in standard position:

(a) $-315°$ (b) $-130°$ (c) $100°$ (d) $730°$

(a)



1.1.3



Types of angles:

* Right angle: Exactly 90°
* Straight angle: Exactly 180°
* Acute angle: Between 0° and 90°
* Obtuse angle: Between 90° and 180°

Complementary angles: Two positive angles that add up to 90°.

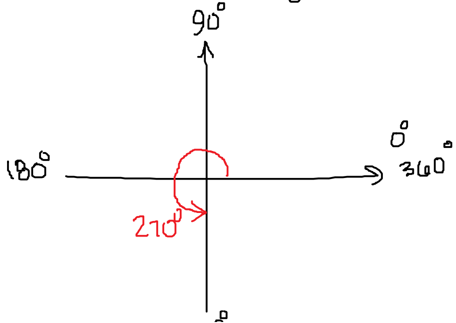
Supplementary angles: Two positive angles that add up to 180°

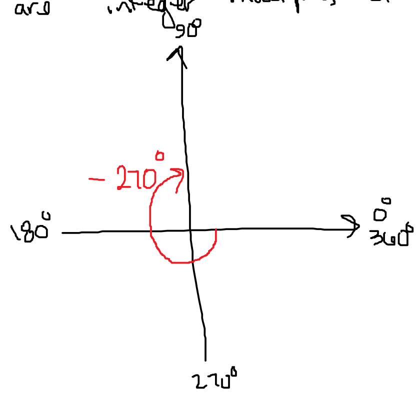
Ex. What angle is the complement of 20°? $\boxed{70^\circ}$ (because $70^\circ + 20^\circ = 90^\circ$)

Ex: What angle is the supplement of 105°? $\boxed{75^\circ}$ (because $75^\circ + 105^\circ = 180^\circ$)

Quadrantal Angles: terminal side lies on the x-axis or y-axis. (when in standard position)

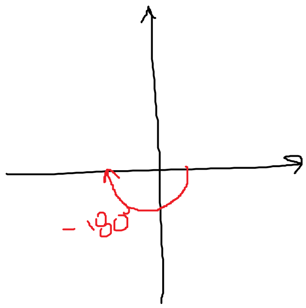
All quadrantal angles are integer multiples of 90°



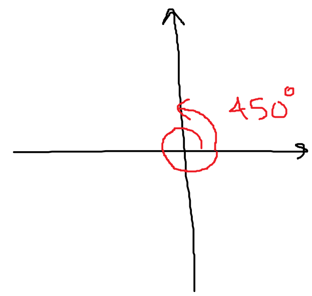


270°

$\boxed{1.1.4}$



$-180°$

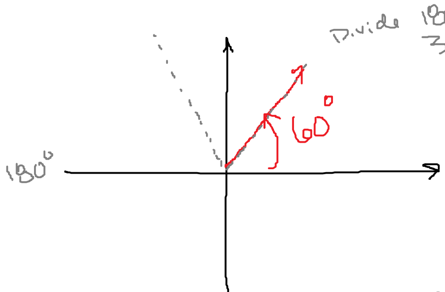


450°

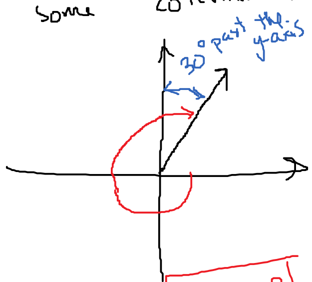
Coterminal Angles: Have the same initial side and terminal side, but different amounts of rotation. Their measures differ by an integer multiple of 360°.

Example: Sketch 60°. Find two positive angles and two negative angles that are coterminal to 60°.

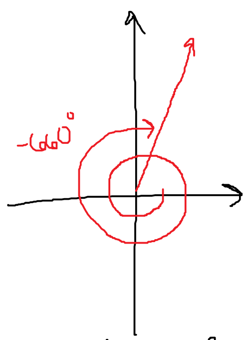
Divide 180° into 3 equal pieces



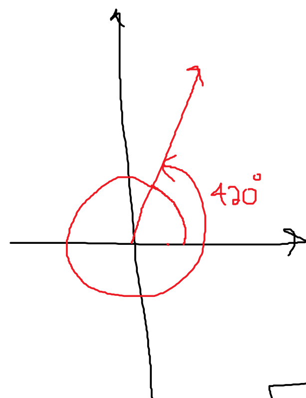
Find some coterminal angles:



$\boxed{-300°}$

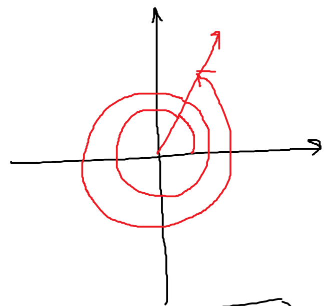


$-300° - 360°$
$= \boxed{-660°}$



$60° + 360° = \boxed{420°}$

1.1.5



$420^\circ + 360^\circ = \boxed{780^\circ}$

$60^\circ + 2(360^\circ) = \boxed{780^\circ}$ (start with 60° and add 2 full revolutions)

To get more: $60^\circ - 9(360^\circ) = \boxed{-3180^\circ}$

$60^\circ + 13(360^\circ) = \boxed{4740^\circ}$

Ex: Write an expression that generates all angles coterminal to 173°.

(there are infinitely many)

$\boxed{173^\circ + 360^\circ n, \text{ where } n \text{ is any integer.}}$

Integers: ..., -3, -2, -1, 0, 1, 2, 3, 4, ...

If $n = -5$, then the coterminal angle is $173^\circ + 360^\circ(-5) = \boxed{-1627^\circ}$

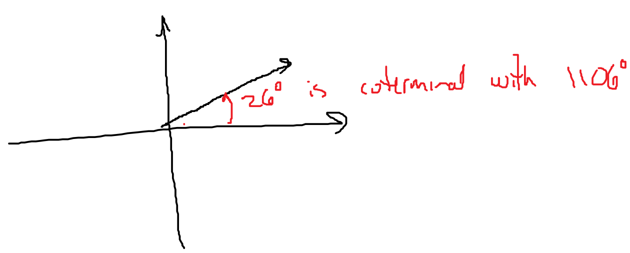
Ex. Find the angle of least positive measure that is coterminal with the given angle.

(a) 1106° (b) -150° (c) -603°

(a) We would do the same thing if we wanted to sketch 1106°

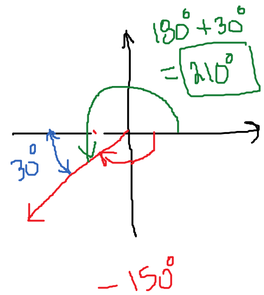
$\frac{1106^\circ}{360^\circ} \approx 3.07$

$1106^\circ - 3(360^\circ) = \boxed{26^\circ}$



1.1.6

(b)



(c) -603°

$\frac{603^\circ}{360^\circ} = 1.675$

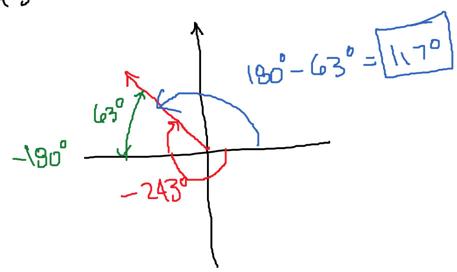
So one full revolution plus some more.

$603^\circ - 360^\circ = 243^\circ$

So -243° is coterminal with -603°

$-603^\circ + 360^\circ = -243^\circ$

$-243^\circ + 360^\circ = \boxed{117^\circ}$



Naming Angles:

We often use Greek letters to represent angles.

θ ("theta")

α ("alpha")

β ("beta")

Degrees, minutes, and Seconds:

1 degree is made up of 60 smaller parts called minutes.

1 minute is made up of 60 smaller parts called seconds.

1 degree = 60 minutes $\iff$ $1^\circ = 60'$

1 minute = $\frac{1}{60}$ degree $\iff$ $1' = \left(\frac{1}{60}\right)^\circ$

1 minute = 60 seconds $\iff$ $1' = 60''$

1 second = $\frac{1}{60}$ minute $\iff$ $1'' = \left(\frac{1}{60}\right)'$

So, how many seconds in 1 degree?

$$1\ \cancel{\text{degree}}\left(\frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{degree}}}\right)\left(\frac{60\text{ seconds}}{1\ \cancel{\text{minute}}}\right) = 60\cdot 60\text{ seconds}$$

$$= \boxed{3600\text{ seconds}}$$

Calculations with Degrees, Minutes, and seconds:

Ex: $28^\circ 35' + 63^\circ 52'$

Do it vertically:

$$\begin{array}{r} 28^\circ 35' \\ +\ 63^\circ 52' \\ \hline 91^\circ 87' \end{array}$$

If there are more than 60 minutes, we need to turn 60 of those minutes into 1 degree.

$$91^\circ 87' = 91^\circ + 60' + 27'$$

$$= 91^\circ + 1^\circ + 27'$$

$$= \boxed{92^\circ 27'}$$

1.1.8

Ex: $73^\circ 23' - 47^\circ 48'$

$$\begin{array}{r} 73^\circ 23' \\ -\ 47^\circ 48' \end{array} \Longrightarrow \begin{array}{r} 72^\circ 83' \\ -\ 47^\circ 48' \\ \hline \boxed{25^\circ 35'} \end{array}$$

can't subtract 48' from 23', so we "borrow" 1 degree and turn it into 60'

Converting between decimal degrees and degrees/minutes/seconds:

Ex: Write $105^\circ 20' 32''$ as decimal degrees. Round to nearest thousandth.

$105^\circ 20' 32''$

$$= 105^\circ + \left(\frac{20}{60}\right)^\circ + \left(\frac{32}{3600}\right)^\circ$$

because $1' = \left(\frac{1}{60}\right)^\circ$ and $1'' = \left(\frac{1}{3600}\right)^\circ$.

$$\approx 105.3422\bar{2}^\circ$$

$$\approx \boxed{105.342^\circ}$$

1.1.9

Ex: Convert 85.263° to degrees, minutes, and seconds.

$$85.263^\circ = 85^\circ + 0.263^\circ\left(\frac{60'}{1^\circ}\right)$$

$$= 85^\circ + 15.78'$$

$$= 85^\circ + 15' + 0.78'$$

$$= 85^\circ + 15' + 0.78'\left(\frac{60''}{1'}\right)$$

$$= 85^\circ + 15' + 46.8''$$

$$= \boxed{85^\circ\ 15'\ 47''}$$ (round to the nearest second)