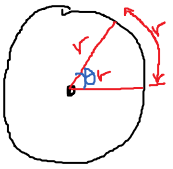
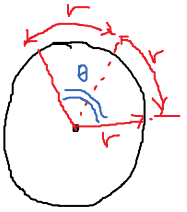
3.1: Radian Measure

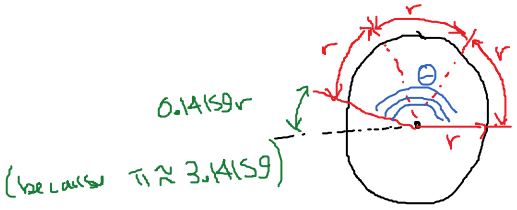
An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius has a radian measure of 1.



Here, angle θ has radian measure of 1.
It cuts off an arc that is 1 radius in length.



This θ has radian measure of 2
It cuts off an arc that is 2 radii in length.



This θ has radian measure of 3

π is the ratio of a circle's circumference to its diameter

$$\pi = \frac{\text{Circumference}}{\text{diameter}} = \frac{\frac{1}{2}\text{ the circumference}}{\text{radius}}$$

A radian measure of π is equal to 180°

$$\pi = 180^\circ$$

(no units) (degrees)

Note: Radian measure is unitless.

Converting between degree measure and radian measure.

Ex: $153\text{ in}\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 12.75\text{ ft}$

Ex: Convert 30° to radian measure.

$$30^\circ = 30^\circ\left(\frac{\pi \text{ (radians)}}{180^\circ}\right) = \frac{30\pi}{180} = \frac{3\pi}{18} = \frac{\pi}{6}$$

$$\boxed{30^\circ = \frac{\pi}{6}}$$

Ex: convert a radian measure of 2.37 to degrees

$$2.37\text{ (radians)}\left(\frac{180^\circ}{\pi\text{ (radians)}}\right) = \left(\frac{426.6}{\pi}\right)^\circ \approx \boxed{135.79^\circ}$$

(use π button on calculator)

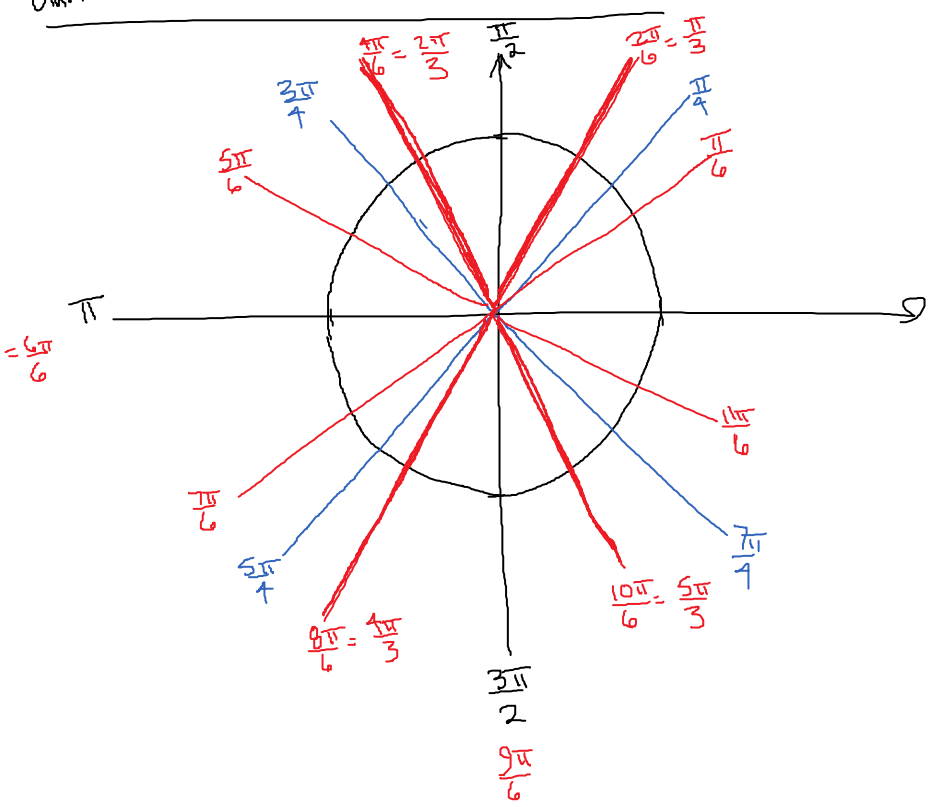
Ex. Convert a radian measure of $\frac{\pi}{12}$ to degrees.

$$\frac{\pi}{12}\text{ (radians)}\left(\frac{180^\circ}{\pi\text{ (radians)}}\right) = \left(\frac{180}{12}\right)^\circ = \boxed{15^\circ}$$

Ex: Convert 217° to radian measure.

$$217^\circ\left(\frac{\pi\text{ (radians)}}{180^\circ}\right) = \boxed{\frac{217\pi}{180}}$$

Unit Circle in Radian Measure



Note: 0 and 2π are coterminal angles

$180^\circ = \pi$

$360^\circ = 2\pi$

$90^\circ = \frac{\pi}{2}$

$45^\circ = \frac{\pi}{4}$

$30^\circ = \frac{\pi}{6}$

$60^\circ = \frac{\pi}{3}$

$270^\circ = \frac{3\pi}{2}$

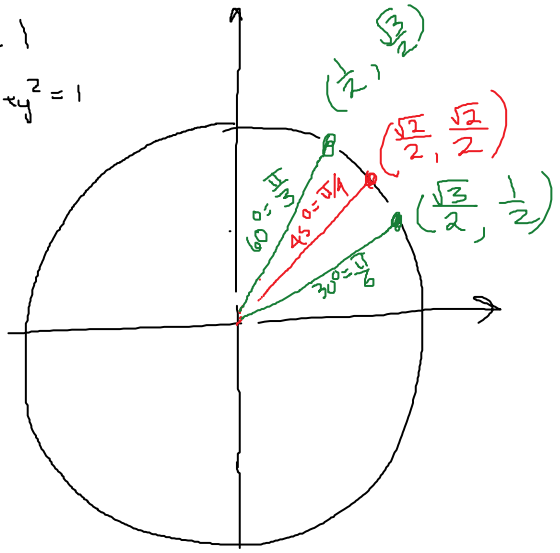
Trig Functions of multiples of $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{2}$

(multiples of $30°, 60°, 45°, 90°$)

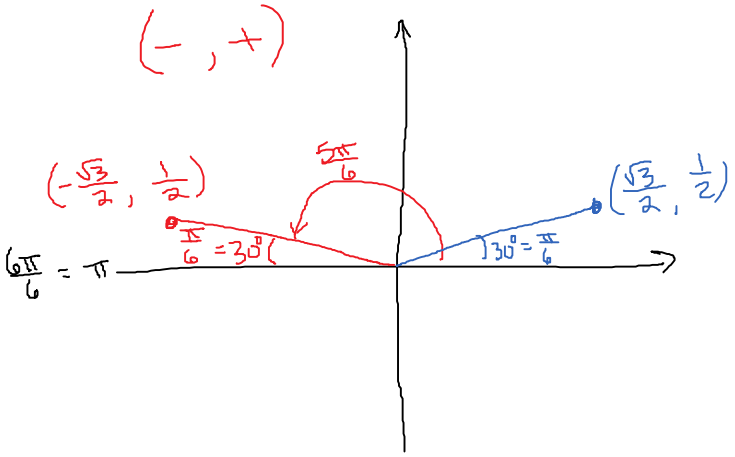
Unit circle:
radius = 1

Equation: $x^2+y^2=1$

On unit circle,
$x=\cos\theta$
$y=\sin\theta$



Ex. Find $\cos\left(\frac{5\pi}{6}\right)$, $\sin\left(\frac{5\pi}{6}\right)$, $\tan\left(\frac{5\pi}{6}\right)$



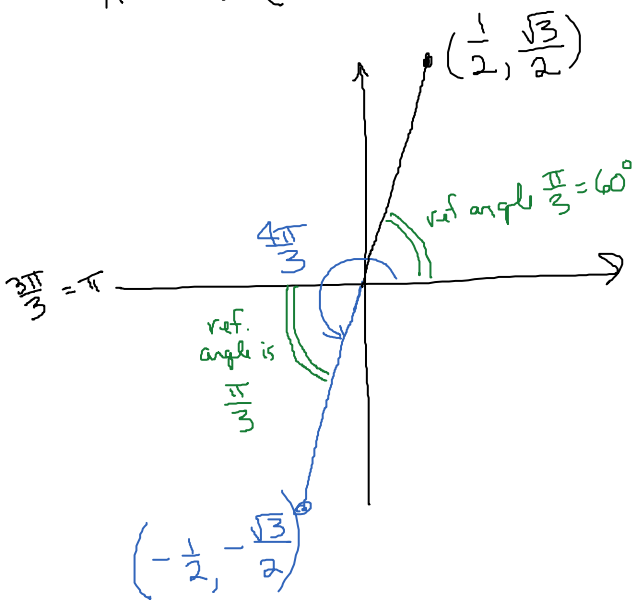
$$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\tan\left(\frac{5\pi}{6}\right) = \frac{y}{x} = \frac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)} = \frac{1/2}{-\sqrt{3}/2}$$

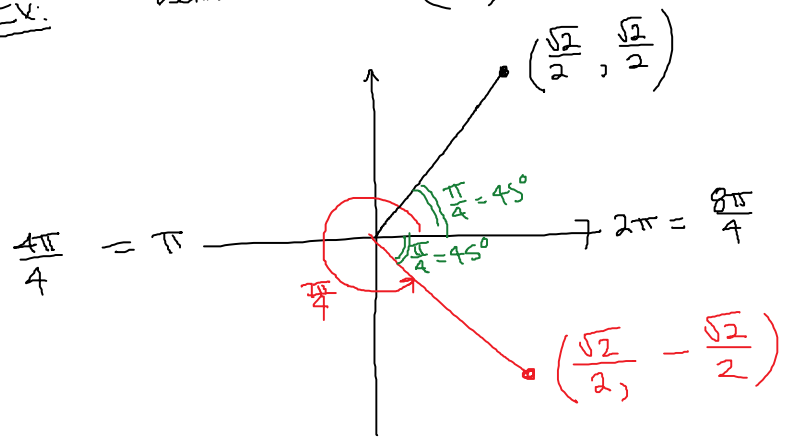
$$= \frac{1}{2}\left(-\frac{2}{\sqrt{3}}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} = \tan\left(\frac{5\pi}{6}\right)$$

Ex: Find $\sin\left(\frac{4\pi}{3}\right)$.



$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Ex: Determine $\tan\left(\frac{\pi}{4}\right)$ and $\sec\left(\frac{\pi}{4}\right)$



$$\tan\left(\frac{\pi}{4}\right) = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

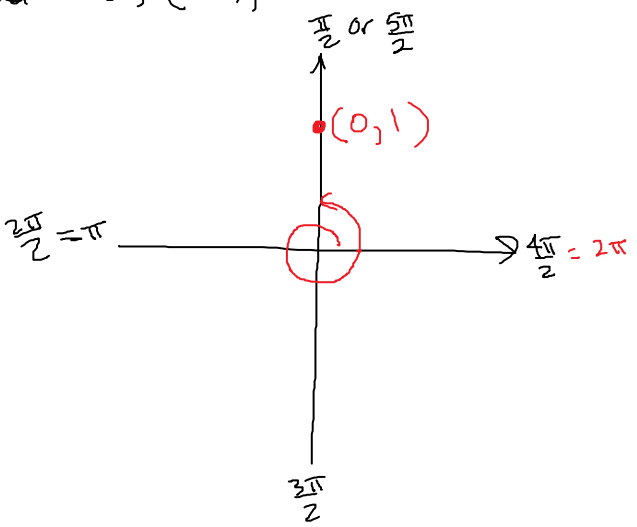
$$\tan\left(\frac{\pi}{4}\right) = -1$$

$$\cos\left(\frac{\pi}{4}\right) = x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{1}$$

$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

Ex. Find $\cos\left(\frac{5\pi}{2}\right)$, $\sin\left(\frac{5\pi}{2}\right)$, $\tan\left(\frac{5\pi}{2}\right)$.



$\frac{\pi}{2} = 90°$

$\cos\left(\frac{5\pi}{2}\right) = 0$

$\sin\left(\frac{5\pi}{2}\right) = 1$

$\tan\left(\frac{5\pi}{2}\right) = \frac{y}{x} = \frac{1}{0}$ undefined

$\tan\left(\frac{5\pi}{2}\right)$ is undefined