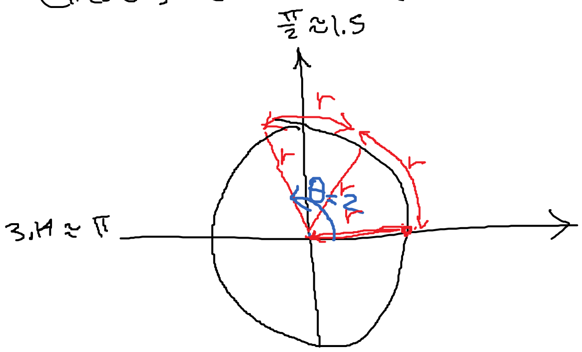
3.2: Applications of Radian Measure
(Arc Length and Sector Area)

Recall: The radian measure of an angle is the number of radii of the circle that can be laid along the circle's circumference (in the portion of the circle cut of by the angle)



$\theta = 2$

★ **Arc Length** ★

The arc length of an intercepted circle arc is equal to:

$s = r\theta$, where r = radius and θ is the radian measure of the angle.

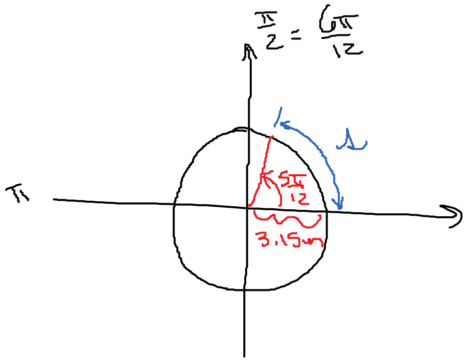
What is the arc length for a whole circle?

All the way around is 2π, so

Circumference $= s = r\theta = r(2\pi)$

$$\boxed{\text{Circumference} = 2\pi r}$$

Ex: what arc length on a circle is intercepted by the angle $\frac{5\pi}{12}$, if the radius is 3.15 meters?



$$s = r\theta = 3.15\text{m}\left(\frac{5\pi}{12}\right)$$

$$= \frac{15.75\pi}{12} \approx \boxed{4.123\text{ m}}$$

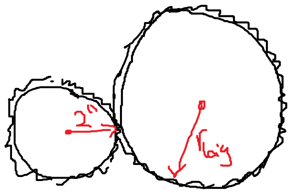
Ex: On a 4-ft radius circle, what arc length is intercepted by an angle of $19°$?

To use $s = r\theta$, we need θ in radian measure.

$$19° = 19\cancel{°}\left(\frac{\pi}{180\cancel{°}}\right) = \frac{19\pi}{180}$$

$$s = r\theta = 4\text{ft}\left(\frac{19\pi}{180}\right) \approx \boxed{1.326\text{ ft}}$$

Ex.:



These are gears. (pretend)

Suppose the smaller gear has radius 2 in.

Suppose that the smaller gear rotates 70° when the big gear rotates 48°. Find the radius of the big gear.

$$s_{\substack{small\\gear}} = s_{\substack{big\\gear}}$$

$$s_{\substack{small\\gear}} = r_{small}\,\theta_{\substack{small\\gear}}$$

$$= 2\text{ in}\cdot 70^\circ$$

$$= 2\text{ in}\cdot 70\cancel{^\circ}\left(\frac{\pi}{180\cancel{^\circ}}\right)$$

$$= \frac{140\pi}{180}\text{ in}$$

$$s_{\substack{big\\gear}} = r_{big}\,\theta_{\substack{big\\gear}}$$

$$= r_{big}\,48^\circ$$

$$= r_{big}\cdot 48\cancel{^\circ}\left(\frac{\pi}{180\cancel{^\circ}}\right)$$

$$= \frac{48\pi}{180}r_{big}$$

$$\frac{140\pi}{180}\text{ in} = \frac{48\pi}{180}r_{big}$$

$$\frac{140\cancel{\pi}}{1\cancel{80}}\text{ in}\cdot\frac{1\cancel{80}}{48\cancel{\pi}} = r_{big}$$

$$\frac{140}{48}\text{ in} = r_{big}$$

$$\frac{70}{24}\text{ in} = r_{big}$$

$$r_{big} = \frac{35}{12}\text{ in} \approx \boxed{2.917\text{ in}}$$

Sector Area

Recall: Area of a circle is $A = \pi r^2$.

What is the area of a circle sector interrupted by angle θ?

All the way around is 2π.

Set up a proportion.

$$\frac{\theta}{2\pi} = \frac{\text{Sector area}}{\pi r^2}$$

$$\frac{\theta}{2\cancel{\pi}} \cdot \cancel{\pi} r^2 = \text{Sector area}$$

$$\frac{\theta r^2}{2} = \text{Sector area}$$

The area of a circle sector cut off by angle θ is

$$A_{\text{sector}} = \frac{1}{2} r^2 \theta,$$

where r is the radius and θ is the radian measure of the angle.

Ex: What is the area of the sector cut off by an angle of $40°$ on a 11 m diameter circle? ~~with radius 5.5 meters?~~

diameter = 11m $\Rightarrow$ radius = $\frac{11\text{m}}{2} = 5.5\text{m}$.

Sector Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(5.5\text{m})^2 \left(\frac{4\pi}{18}\right)$

$= \boxed{\frac{121\pi}{36}\ \text{m}^2}$

$\approx \boxed{10.56\ \text{m}^2}$

Need to get $\theta = 40°$ into radian measure.

$$\theta = 4\cancel{0}°\left(\frac{\pi}{18\cancel{0}}\right) = \frac{40\pi}{180} = \frac{4\pi}{18}$$