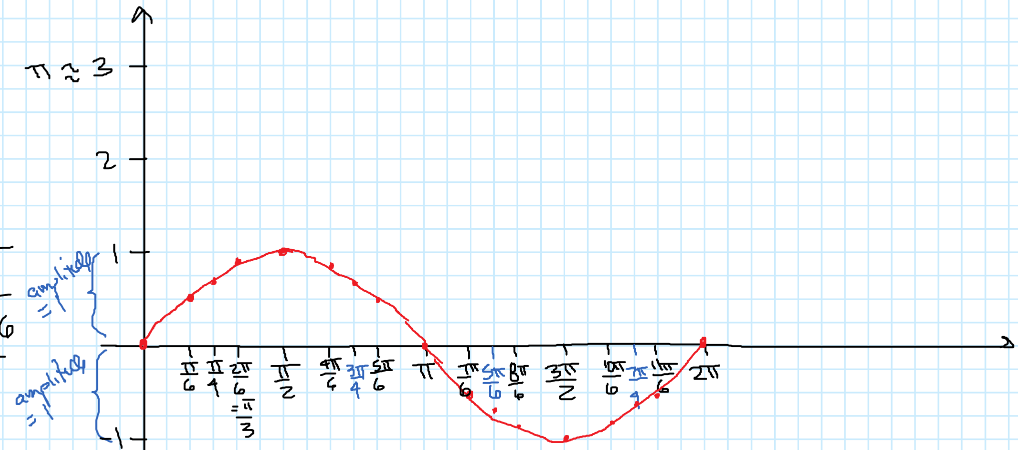
Monday, October 14, 2019 1:24 PM

Wednesday, October 9, 2019
1:08 PM

4.1: Graphs of the Sine and Cosine Functions

Let's graph ~~$f(x) = \sin(x)$~~
$f(\measuredangle) = \sin(\measuredangle)$ same as $f(\theta) = \sin(\theta)$

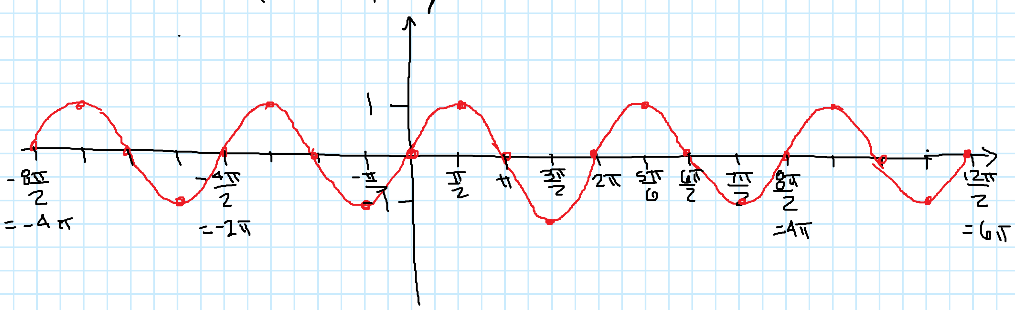
θ	$\sin\theta$
0	$\sin(0) = 0$
$\frac{\pi}{6} = 30^\circ$	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = 0.5$
$\frac{\pi}{4} = 45^\circ$	$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \approx 0.707$
$\frac{\pi}{3} = 60^\circ$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \approx 0.866$
$\frac{\pi}{2} = 90^\circ$	$\sin\left(\frac{\pi}{2}\right) = 1$
$\frac{2\pi}{3} = 120^\circ$	$\frac{\sqrt{3}}{2} \approx 0.866$
$\frac{3\pi}{4} = 135^\circ$	$\frac{\sqrt{2}}{2} \approx 0.707$
$\frac{5\pi}{6} = 150^\circ$	$\frac{1}{2} = 0.5$
π	0
$\frac{7\pi}{6}$	$-\frac{1}{2} = -0.5$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2} \approx -0.707$
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \approx -0.866$
$\frac{3\pi}{2}$	-1



This is 1 cycle (1 period) of the function $f(\measuredangle) = \sin(\measuredangle)$

or, equivalently $f(\theta) = \sin\theta$
$f(x) = \sin(x)$

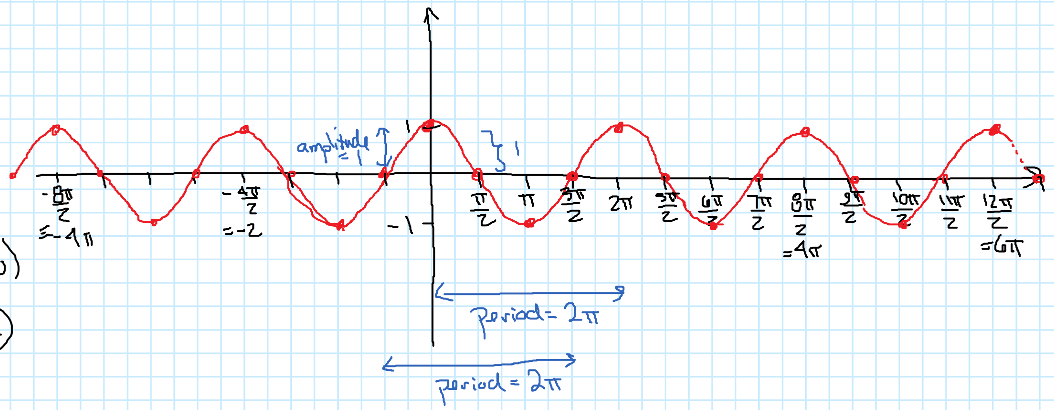
It keeps repeating



Let's graph $y = \cos(x)$ or equivalently, $f(x) = \cos(x)$

$f(\theta) = \cos(\theta)$ or $f(\measuredangle) = \cos(\measuredangle)$

θ	$f(\theta) = \cos(\theta)$
0	$\cos(0) = 1 \Rightarrow (0, 1)$
$\frac{\pi}{2}$	$\cos\left(\frac{\pi}{2}\right) = 0 \Rightarrow \left(\frac{\pi}{2}, 0\right)$
π	$\cos(\pi) = -1 \Rightarrow (\pi, -1)$
$\frac{3\pi}{2}$	$\cos\left(\frac{3\pi}{2}\right) = 0 \Rightarrow \left(\frac{3\pi}{2}, 0\right)$
2π	$\cos(2\pi) = 1 \Rightarrow (2\pi, 1)$



A function is called a periodic function if there is a number p such that $f(x+p) = f(x)$ for every x in the domain of the function.

The smallest such p is called the period of the function.

The sine and cosine functions have period 2π.

The amplitude of a function is $A = \frac{1}{2}(y_{max} - y_{min})$, if y_{max} and y_{min} are finite numbers

For sine and cosine, $y_{max} = 1$ and $y_{min} = -1$,

so amplitude is $A = \frac{1}{2}(1-(-1)) = \frac{1}{2}(1+1) = \frac{1}{2}(2) = 1$

(Amplitude = half the height)

Transformations of the sine and cosine functions

Changing the period and the amplitude.

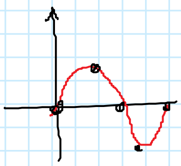
The graph of $f(x) = A\sin(Bx)$ or $g(x) = A\cos(Bx)$ has amplitude $|A|$ and period $\frac{2\pi}{B}$

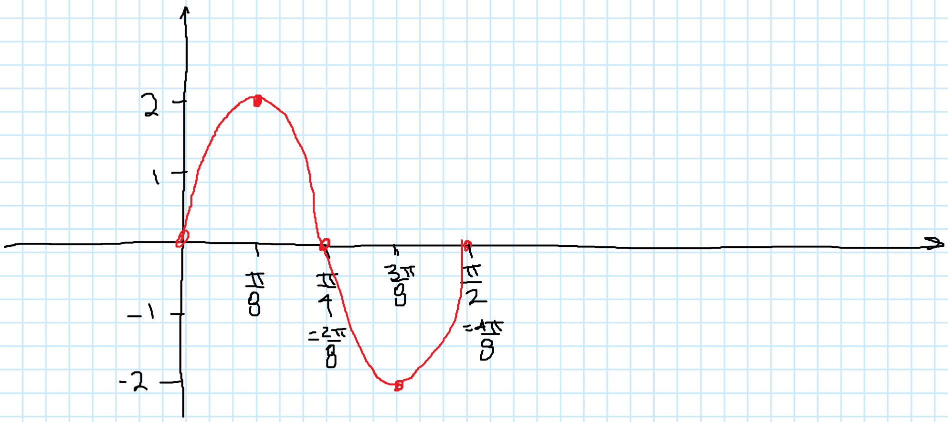
$$\left(\text{new period} = \frac{\text{old period}}{\text{coefficient of angle}}\right)$$

Ex: Graph $y = 2\sin(4x)$.

Amplitude $= 2$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$ Divide into 4 equal pieces

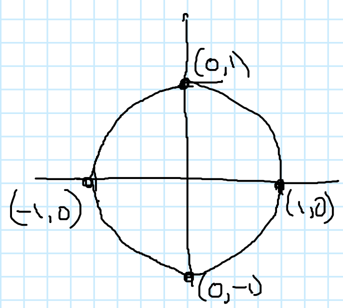


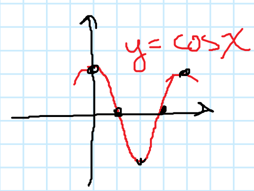


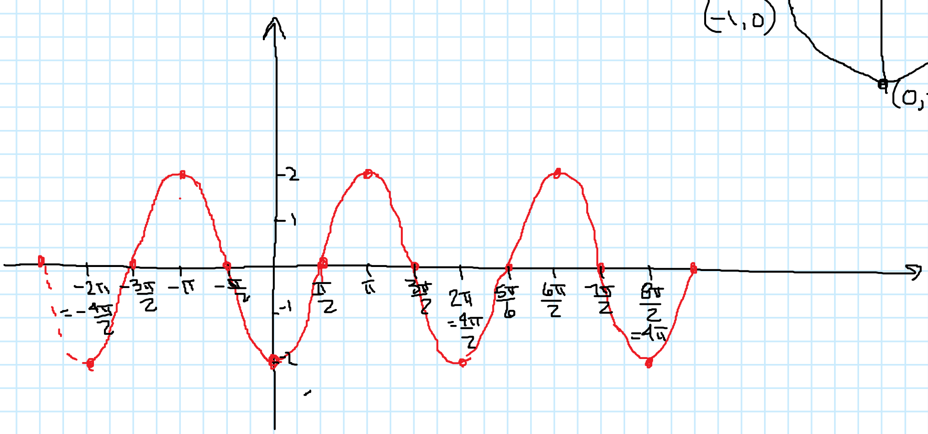
Ex: Graph $y = -2\cos(x)$. Graph 3 periods.

Amplitude: $|-2| = 2$

Period = length of 1 cycle: $0 \le x < 2\pi$







Ex: Graph $f(x) = -3\sin(4x)$.

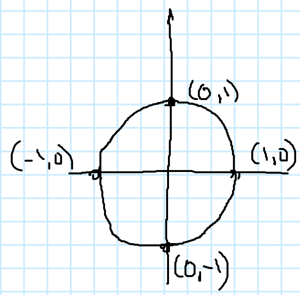
Amplitude: $|-3| = 3$

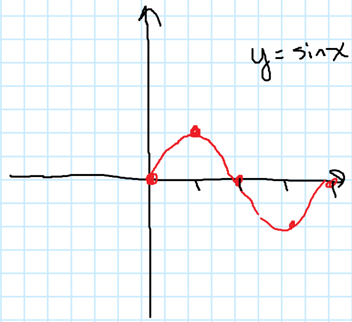
Period: 1 Period of $y = \sin x$: $0 \le x \le 2\pi$

1 Period of $y = \sin(4x)$: $0 \le 4x \le 2\pi$

$$\frac{0}{4} \le \frac{4x}{4} \le \frac{2\pi}{4}$$

$$0 \le x \le \frac{\pi}{2}$$

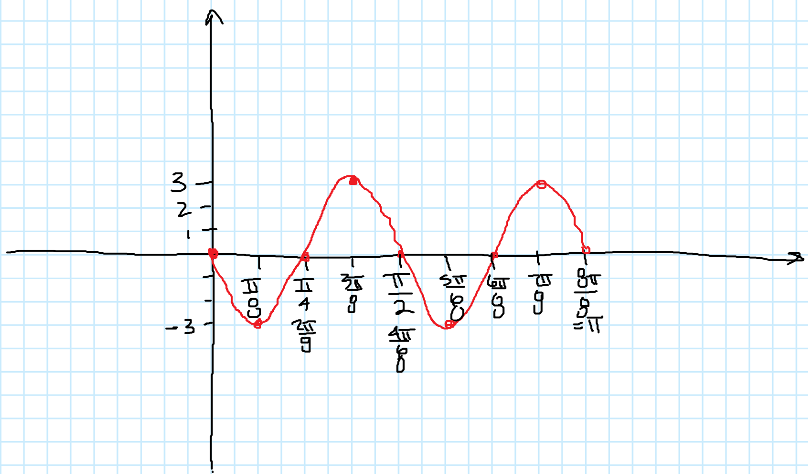




$$1 \text{ period} = \frac{\text{old period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$\boxed{\text{Period: } \frac{\pi}{2} \quad \text{Amplitude: } 3}$

The minus sign in front reflects it around the x-axis



Ex: Graph $g(x) = \frac{1}{2}\cos\left(\frac{\pi x}{3}\right)$

Amplitude: $A = \left|\frac{1}{2}\right| = \frac{1}{2}$

Period $\frac{2\pi}{B}$ where B is coefficient of x

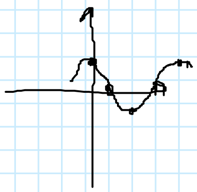
$\frac{2\pi}{\pi/3} = \frac{2\cancel{\pi}}{1}\cdot\frac{3}{\cancel{\pi}} = 6$

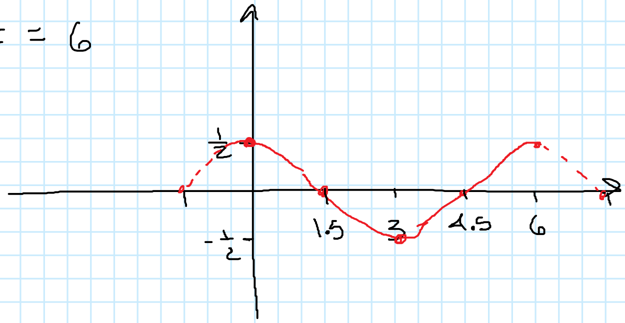
1 period of $y = \cos x$ is $0 \le x \le 2\pi$

1 period of $y = \cos\left(\frac{\pi x}{3}\right)$ is $0 \le \frac{\pi x}{3} \le 2\pi$

$\frac{3}{\pi}(0) \le \frac{\pi x}{3}\left(\frac{3}{\pi}\right) \le 2\pi\left(\frac{3}{\pi}\right)$

$0 \le x \le 6$





Facts about graph of $f(x) = \sin x$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

y-intercept: 0

x-intercepts: $0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots$

$k\pi$, where k is any integer

Symmetry: Symmetric around the origin, so it is an odd function.

Facts about $g(x) = \cos x$

Domain: $(-\infty)$

Range: $[-1, 1]$

y-intercept: 1

x-intercepts: $\pm\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \ldots$

(odd multiples of $\frac{\pi}{2}$)

$(2k+1)\frac{\pi}{2}$, where k is any integer.

Symmetry: symmetric around y-axis, so it is an even function

Both functions are continuous on their whole domain (can be drawn without lifting your pencil)