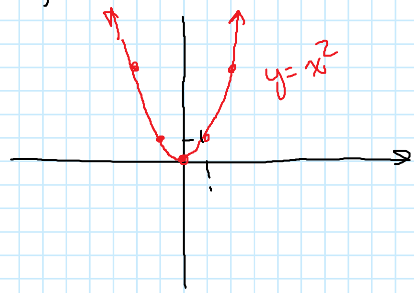
4.2: Translations (Vertical and Horizontal Shifts) of the Sine and Cosine Functions

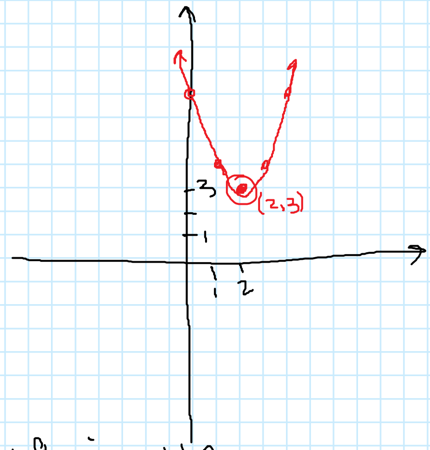
Recall:

Example: Graph $y = (x-2)^2 + 3$

Basic / Parent function: $y = x^2$

x	$y = x^2$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$
-1	$(-1)^2 = 1$
(-2)	$(-2)^2 = 4$
± 3	$(\pm 3)^2 = 9$





Start with $y = x^2$ and then shift it right 2 and up 3

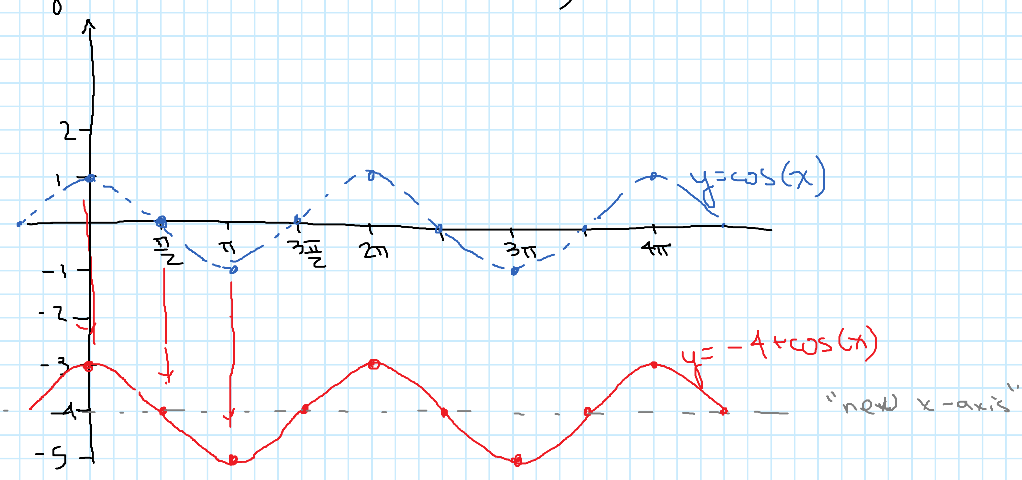
Note: $y - 3 = (x-2)^2$

$y - 3 = 0 \Downarrow y = 3$ (up 3)

$x - 2 = 0 \Downarrow x = 2$ (right 2)

Ex. Graph $y = -4 + \cos(x)$ over 2 periods

Note: $y = -4 + \cos(x)$ is equivalent to
$y = \cos(x) - 4$ (shifts it down 4)



Ex. Graph $y = 3 - 2\sin(3x)$

1^{st} graph $y = -2\sin(3x)$, then shift it up 3.

Amplitude: 2

x-axis reflection.

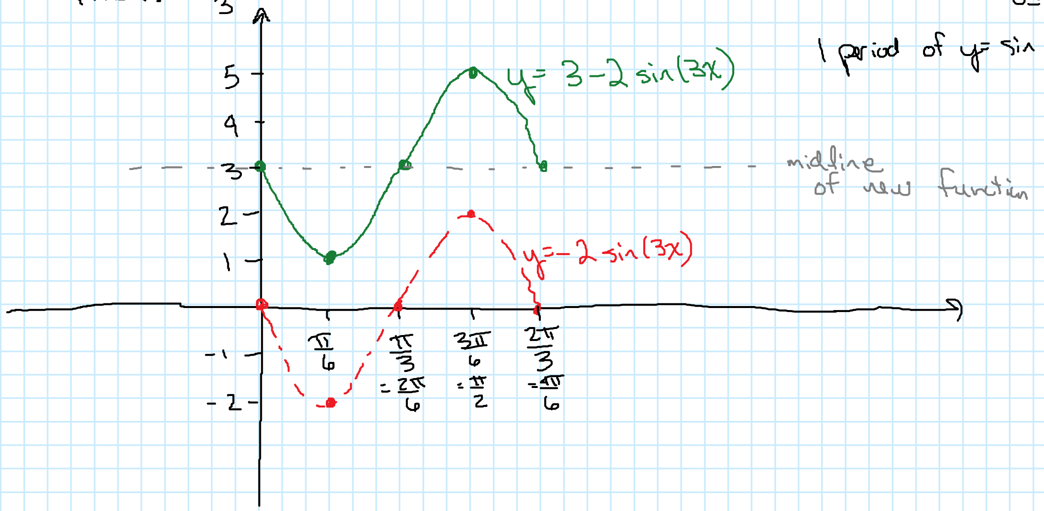
Period: $\frac{2\pi}{3}$

Note: 1 period of $y = \sin x$ is $0 \le x \le 2\pi$

1 period of $y = \sin(3x)$ is $0 \le 3x \le 2\pi$

$\frac{0}{3} \le \frac{3x}{3} \le \frac{2\pi}{3}$

$0 \le x < \frac{2\pi}{3}$



Horizontal Translations: Graphing $f(x) = \sin(x - D)$ or $y = \cos(x - D)$

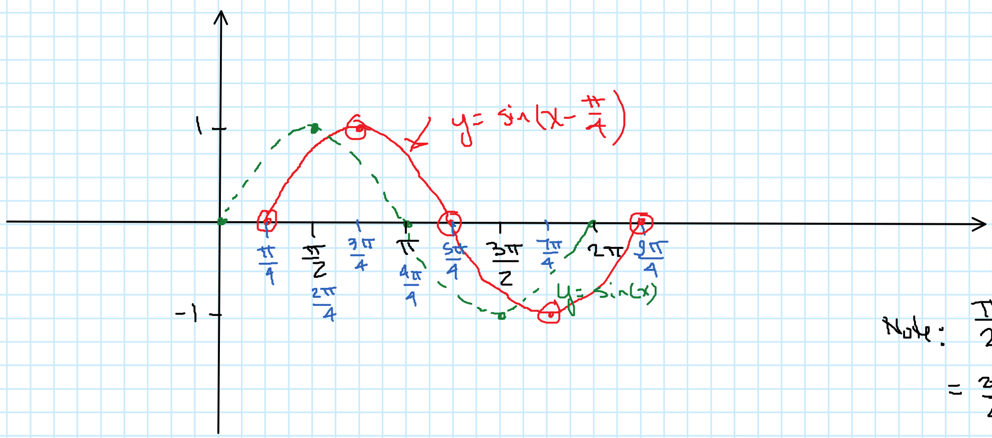
To graph $y = (x-h)^2$, (for $h > 0$) start with $y = x^2$ and shift it right h units

To graph $y = (x+h)^2$ (for $h > 0$), start with $y = x^2$ and shift it left h units

Same for trig functions!

Ex. Graph $y = \sin(x - \frac{\pi}{4})$ over 1 period.

Amplitude: 1
Period: 2π
Vertical shift: none
Horizontal Shift (Phase Shift): right $\frac{\pi}{4}$
For periodic functions, the horizontal shift is called a phase shift.
Reflection: none



Note: $\frac{\pi}{2} + \frac{\pi}{4}$ max

$= \frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$

Ex: Graph $y = 5\cos\left(2x + \frac{2\pi}{3}\right)$

$$\frac{2\pi/3}{2} = \frac{2\pi}{3}\cdot\frac{1}{2} = \frac{2\pi}{6} = \frac{\pi}{3}$$

Rewrite: $y = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

1st graph $y = 5\cos(2x)$ then shift it left $\frac{\pi}{3}$

(because we replaced x by $x + \frac{\pi}{3}$)

Amplitude 5

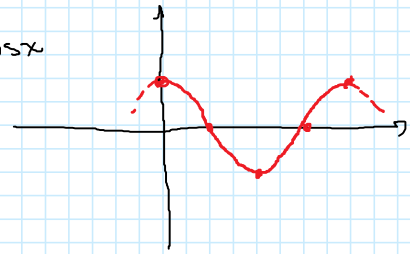
Period: $\frac{2\pi}{2} = \pi$

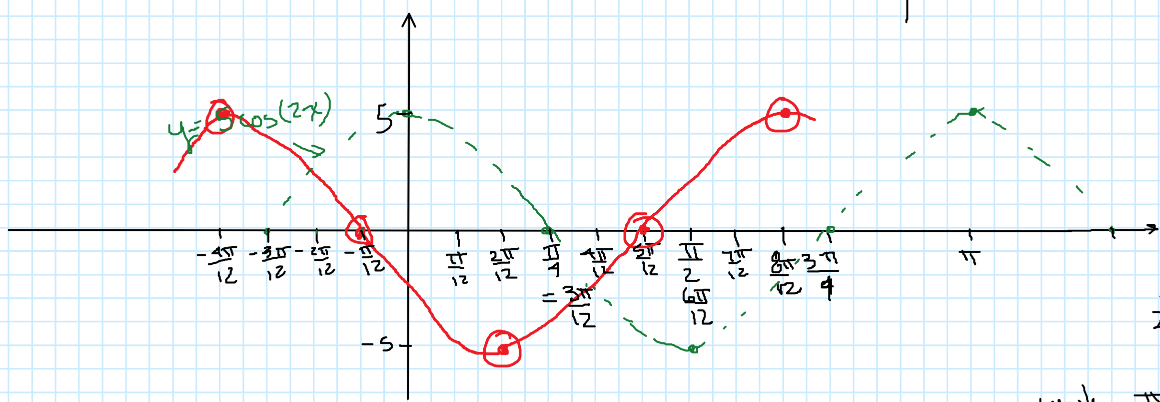
(Period of $y = \cos Bx$ is $\frac{2\pi}{B}$)

Vertical Shift: none

Reflection: none

$y = \cos x$





Period $= \pi$

$\frac{1}{4}$ of a period is $\frac{\pi}{4}$

write $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with common denominator

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

1st point: $0 \longrightarrow -\frac{\pi}{3} = -\frac{4\pi}{12}$

2nd: $\frac{\pi}{4} \longrightarrow \frac{\pi}{4} - \frac{\pi}{3} = \frac{3\pi}{12} - \frac{4\pi}{12} = -\frac{\pi}{12}$

3rd: $\frac{\pi}{2} \longrightarrow \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6} = \frac{2\pi}{12}$

4th: $\frac{3\pi}{4} \longrightarrow \frac{3\pi}{4} - \frac{\pi}{3} = \frac{9\pi}{12} - \frac{4\pi}{12} = \frac{5\pi}{12}$

5th: $\pi \longrightarrow \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3} = \frac{8\pi}{12}$

If $\frac{\pi}{12}$ is 2 squares, then $\frac{\pi}{3} = \frac{4\pi}{12} = 8$ squares