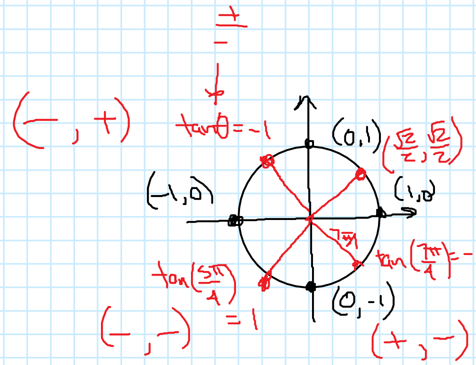
4.3: Graphs of the Tangent and Cotangent Functions

Graph of $y = \tan(x)$

x	$\cos(x)$	$\sin(x)$	$y=\tan(x) = \frac{\sin(x)}{\cos(x)}$
0	1	0	$\frac{0}{1} = 0$
$\frac{\pi}{2}$	0	1	$\frac{1}{0}$ undef.
π	-1	0	$\frac{0}{-1} = 0$
$\frac{3\pi}{2}$	0	-1	$\frac{-1}{0}$ undefined
2π	1	0	$\frac{0}{1} = 0$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$
$\frac{3\pi}{4}$			-1
$\frac{5\pi}{4}$			1
$\frac{7\pi}{4}$			-1
$-\frac{\pi}{2}$	0	-1	$\frac{-1}{0}$ undefined

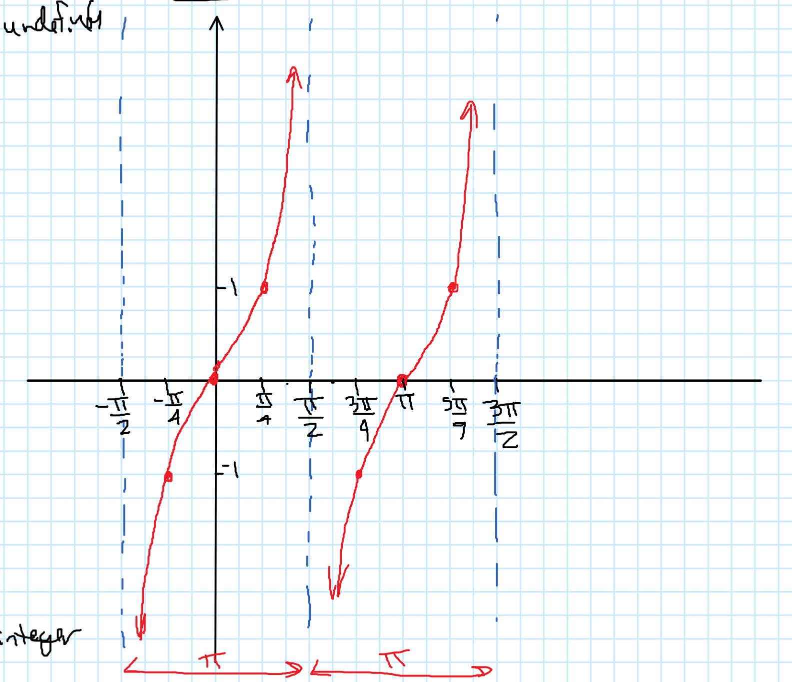


Notice: $y = \tan(x)$ repeats after π instead of after 2π

$\boxed{y = \tan(x) \text{ and } y = \cot(x) \text{ have period } \pi}$

$\boxed{y = \sin(x),\ y = \cos(x),\ y = \csc(x),\ y = \sec(x) \text{ all have period } 2\pi}$

Graph of $y = \tan x$:



Note: "amplitude" is not meaningful for $y=\tan(x)$ and $y=\cot(x)$

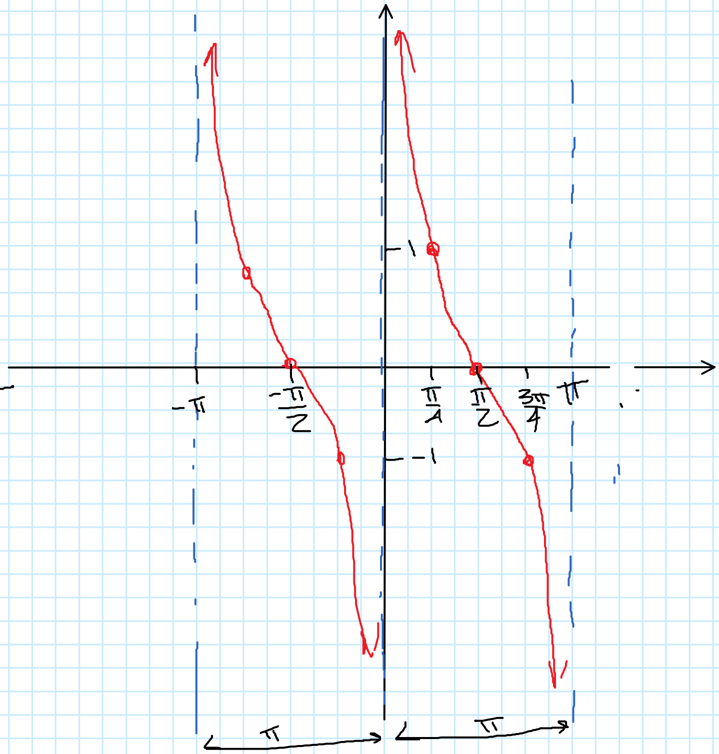
Range: $(-\infty, \infty)$

Domain: All real numbers except $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$

$$x \neq \frac{(2k+1)\pi}{2}, \quad k \text{ any integer}$$

(odd multiples of $\frac{\pi}{2}$)

Graph of $y = \cot(x)$



Range: $(-\infty, \infty)$

Domain: all real numbers except integer multiples of π

$x \neq k\pi$, where k is any integer

Example. Graph $y = \tan(2x)$.

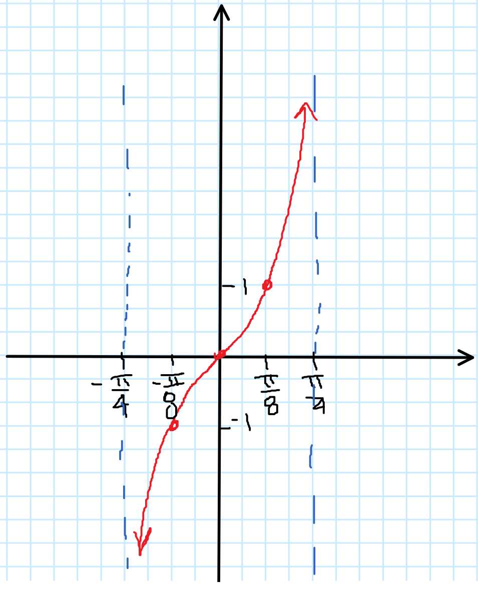
Note: $y = \tan(Bx)$ and $y = \cot(Bx)$ have period $\frac{\pi}{B}$. $\left(\frac{\text{old period}}{\text{multiplier}}\right) = \text{new period}$

1 period of $y = \tan(x)$ is: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

equivalent period of $y = \tan(2x)$ is $-\frac{\pi}{2} < 2x < \frac{\pi}{2}$

$-\frac{\pi}{2} \cdot \left(\frac{1}{2}\right) < \left(\frac{1}{2}\right) 2x < \frac{\pi}{2}\left(\frac{1}{2}\right)$

$-\frac{\pi}{4} < x < \frac{\pi}{4}$



For $y = \tan(2x)$, $B = 2$

New period is $\frac{\pi}{2}$, so $\frac{\pi}{4}$ on each side.

4.3.3

Ex: Graph $y = 3\cot\left(\frac{\pi}{2}x\right)$

1 period of $y = \cot(x)$: $0 < x < \pi$

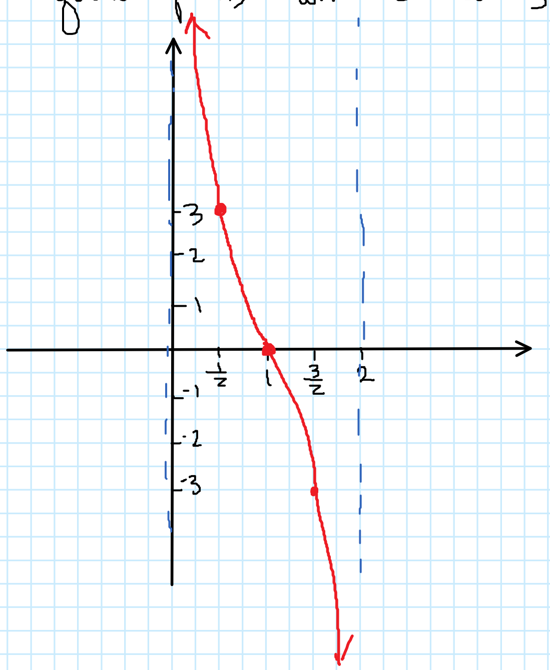
equivalent period of $y = \cot\left(\frac{\pi}{2}x\right)$: $0 < \frac{\pi}{2}x < \pi$

$$0\left(\frac{2}{\pi}\right) < \left(\frac{2}{\pi}\right)\frac{\pi}{2}x < \pi\left(\frac{2}{\pi}\right)$$

$$0 < x < 2$$

OR, Note that $\frac{\pi}{B} = \frac{\pi}{\frac{\pi}{2}} = \pi \cdot \frac{2}{\pi} = 2$

The 3 in front will multiply the y-values by 3, so the quarter-points will be at 3 and -3 (instead of 1 and -1)



Ex: Graph $y = 1 - 2\cot(2x + \pi)$

Rewrite: $y = -2\cot\left(2\left(x + \frac{\pi}{2}\right)\right) + 1$

Start with $y = \cot(2x)$ then
- shift it left $\frac{\pi}{2}$
- reflect it around x-axis
- shift it up 1
- multiply the y-values (quarter-points) by 2

Figure out period of $y = \cot(2x)$

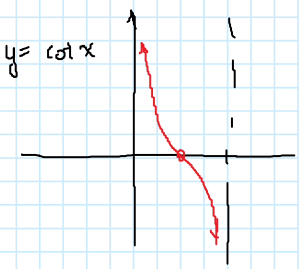
1 period of $y = \cot(x)$: $0 < x < \pi$

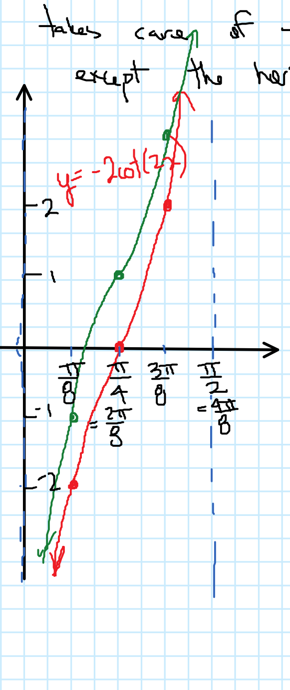
1 period of $y = \cot(2x)$: $0 < 2x < \pi$

$\frac{0}{2} < \frac{2x}{2} < \frac{\pi}{2}$

$0 < x < \frac{\pi}{2}$

Graph $y = -2\cot(2x)$ takes care of everything except the horizontal & vertical shifts.





Horizontal shift: left $\frac{\pi}{2}$

Vertical shift: up 1

Horizontal shift left $\frac{\pi}{2}$ moves it left to overlap exactly with previous period. So nothing ch s.

Ex. Graph $y = \frac{1}{2}\tan\left(3x + \frac{\pi}{2}\right)$

$$y = \frac{1}{2}\tan\left(3\left(x + \frac{\pi}{6}\right)\right)$$

Note: $\frac{\frac{\pi}{2}}{3} = \frac{\pi}{2}\cdot\frac{1}{3} = \frac{\pi}{6}$

Start with $y = \tan(x)$.

Multiply the y-values (quarter points by $\frac{1}{2}$)

Period becomes: $\frac{\pi}{B} = \frac{\pi}{3}$

half on each side, so

$\frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$ on each side

$y = \tan(x)$: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$y = \tan(3x)$ $-\frac{\pi}{2} < 3x < \frac{\pi}{2}$

$-\frac{\pi}{2}\left(\frac{1}{3}\right) < \left(\frac{1}{3}\right)3x < \frac{\pi}{2}\left(\frac{1}{3}\right)$

$-\frac{\pi}{6} < x < \frac{\pi}{6}$