# 5.1: Fundamental Identities

5.1.1

## Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Note: $\cos^2\theta + \sin^2\theta = 1$

Divide by $\cos^2\theta$:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

Start over with

$$\cos^2\theta + \sin^2\theta = 1$$

Divide by $\sin^2\theta$:

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

## Reciprocal Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ (also $\sin\theta = \dfrac{1}{\csc\theta}$)

$\sec\theta = \dfrac{1}{\cos\theta}$ (also $\cos\theta = \dfrac{1}{\sec\theta}$)

$\cot\theta = \dfrac{1}{\tan\theta}$ (also $\tan\theta = \dfrac{1}{\cot\theta}$)

## Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

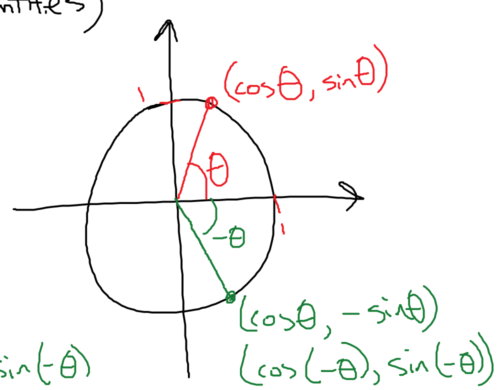
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

## Opposite-angle Identities (Even-odd identities)

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\tan(-\theta) = -\tan\theta$$




Note: $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)}$

$$= \frac{-\sin\theta}{\cos\theta} = -\tan(\theta)$$

• Ex: Suppose $\cos\theta = -\frac{3}{8}$

Find $\cos(-\theta)$.

$$\boxed{\cos(-\theta) = -\frac{3}{8}}$$

Ex: Suppose $\sin\theta = -0.8$.

Find $\sin(-\theta)$.

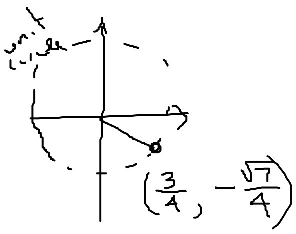
$$\sin(-\theta) = -\sin\theta = -(-0.8)$$
$$= \boxed{+0.8 = \sin(-\theta)}$$

Ex: Suppose $\cos\theta = \frac{3}{4}$ and $\theta$ is in Quadrant IV.

Find $\sin\theta$, $\sin(-\theta)$, $\tan(\theta)$, $\tan(-\theta)$, $\cos(-\theta)$.

Method 1. Pythagorean Identity.

$$\cos^2\theta + \sin^2\theta = 1$$
$$\left(\frac{3}{4}\right)^2 + \sin^2\theta = 1$$
$$\frac{9}{16} + \sin^2\theta = 1$$
$$\sin^2\theta = 1 - \frac{9}{16}$$
$$(\sin\theta)^2 = \frac{7}{16}$$
$$\sin\theta = \pm\sqrt{\frac{7}{16}} = \pm\frac{\sqrt{7}}{\sqrt{16}}$$
$$= \pm\frac{\sqrt{7}}{4}$$




Quadrant 4, so $\sin\theta$ is negative

$$\boxed{\sin\theta = -\frac{\sqrt{7}}{4}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{7}}{4}}{\frac{3}{4}} = \boxed{-\frac{\sqrt{7}}{3}}$$

Method 2: make a triangle

$\cos\theta = \frac{3}{4}$, Q4

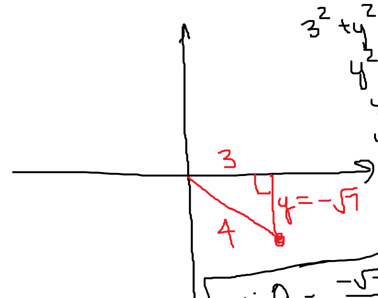
$3^2 + y^2 = 4^2$

$y^2 = 16 - 9$

$y^2 = 7$

$y = \pm\sqrt{7}$

choose $y = -\sqrt{7}$




$$\boxed{\sin\theta = \frac{-\sqrt{7}}{4}}$$

$$\boxed{\tan\theta = \frac{opp}{adj} = \frac{-\sqrt{7}}{3}}$$

$$\sin(-\theta) = -\sin\theta = \boxed{\frac{\sqrt{7}}{4} = \sin(-\theta)}$$

$$\cos(-\theta) = \cos\theta = \boxed{\frac{3}{4} = \cos(-\theta)}$$

$$\tan(-\theta) = -\tan\theta = \boxed{\frac{\sqrt{7}}{3} = \tan(-\theta)}$$

# Simplifying and Factoring Trig Expressions

Ex. Multiply out.

$\cos\theta(\cos\theta - \sin\theta + 2)$

$= \boxed{\cos^2\theta - \cos\theta\sin\theta + 2\cos\theta}$

Simplify.

Ex. $2\sec\theta + \dfrac{5}{\cos\theta} + \tan\theta + \dfrac{1}{\cot\theta}$

$= 2\sec\theta + 5\left(\dfrac{1}{\cos\theta}\right) + \tan\theta + \tan\theta$

$= 2\sec\theta + 5\sec\theta + 2\tan\theta$

$= \boxed{7\sec\theta + 2\tan\theta}$

Ex: Multiply out

$(\sin\theta + \cos\theta)^2$

$= (\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$

$= \sin^2\theta + \sin\theta\cos\theta + \cos\theta\sin\theta + \cos^2\theta$

$= \sin^2\theta + 2\cos\theta\sin\theta + \cos^2\theta$

$= \underbrace{(\sin^2\theta + \cos^2\theta)}_{1} + 2\cos\theta\sin\theta$

$= \boxed{1 + 2\cos\theta\sin\theta}$

Ex: Factor.

$\sin^2\alpha - 1$

$= (\sin\alpha)^2 - 1^2$

$= \boxed{(\sin\alpha + 1)(\sin\alpha - 1)}$

Recall: $a^2 - b^2$

$= (a-b)(a+b)$

Ex: $x^2 - 49$

$= (x+7)(x-7)$

Ex: $y^2 - 1 = (y+1)(y-1)$