5.2: Verifying Trig Identities

Identity: A statement of equality that is true for all values of the variable(s).

We will prove or verify identities.

How to prove an identity.

To prove (left side) = (right side)

Proof should look like: (one side) = ~~~~

= ~~~~

= ~~~~

= ~~~~

⋮

= ~~~~

= (other side)

Example: Prove.

$$\frac{\cot\theta}{\csc\theta} = \cos\theta$$

$$\frac{\cot\theta}{\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\cos\theta}{\cancel{\sin\theta}_{1}} \cdot \frac{\cancel{\sin\theta}^{1}}{1}$$

$$= \frac{\cos\theta}{1}$$

$$= \cos\theta \quad \blacksquare$$

$\blacksquare$ at the end means "It is proved"

Ex: Prove.

$$\cot\theta + \tan\theta = \sec\theta \csc\theta$$

$$\cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

LCD: $\sin\theta\cos\theta$

$$= \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right) + \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\sin\theta}\right)$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$$

$$= \sec\theta\csc\theta \qquad \blacksquare$$

Prove these identities.

$$\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$$

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$$

$$\frac{\tan x}{1+\cos x} + \frac{\sin x}{1-\cos x} = \cot x + \sec x \csc x$$

<u>Note:</u> Pythagorean Identity $\cos^2\theta + \sin^2\theta = 1$

can be written as:

$$\boxed{\begin{aligned} \cos^2\theta &= 1 - \sin^2\theta \\ \text{Also: } \sin^2\theta &= 1 - \cos^2\theta \end{aligned}}$$

Same with the other Pythagorean Identities:

$$1 + \tan^2\theta = \sec^2\theta$$

gives us: $\tan^2\theta = \sec^2\theta - 1$

Also $\cot^2\theta + 1 = \csc^2\theta$

gives us: $\cot^2\theta = \csc^2\theta - 1$

<u>Note:</u> Recall: $a^2 - b^2$
$= (a+b)(a-b)$

So, $1 - \cos^2\theta$
$= (1+\cos\theta)(1-\cos\theta)$

$1 - \sin^2\theta$
$= (1-\sin\theta)(1+\sin\theta)$

Tip: When proving identies involving fractions:
If you have $1+\cos\theta$, try multiplying by $1-\cos\theta$
If you have $1-\sin\theta$, try multiplying by $1+\sin\theta$

Prove these identities:

1) $\dfrac{1-\sin^2\theta}{\cos\theta} = \cos\theta$

2) $\cos^2\theta(\tan^2\theta + 1) = 1$

3) $\dfrac{1+\cos\theta}{\tan^2\theta} = \dfrac{\cos\theta}{\sec\theta - 1}$

4) $\dfrac{1-\sin\theta}{1+\sin\theta} = \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$

#4) Prove.

$$\frac{1-\sin\theta}{1+\sin\theta} = \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

$$\frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{1-\sin\theta-\sin\theta+\sin^2\theta}{1-\sin^2\theta} \quad \longleftarrow \quad (A+B)(A-B) = A^2-B^2$$

$$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - 2\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) + \tan^2\theta$$

$$= \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$