5.3 and 5.4: Sum and Difference Identities

Be able to use these identities:

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

No need to memorize these!

Ex: Find the exact value of $\sin(15°)$.

Use $\sin(A-B) = \sin A \cos B - \cos A \sin B$ with $A = 60°$, $B = 45°$

$$\sin(15°) = \sin(60° - 45°)$$
$$= \sin(60°)\cos(45°) - \cos(60°)\sin(45°)$$
$$= \frac{\sqrt{3}}{2}\cdot\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

Use calculator to check:

$$\frac{\sqrt{6}-\sqrt{2}}{4} \approx 0.2588$$
$$\sin(15°) \approx 0.2588$$

equal! ✓

Ex: Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

Could change to degrees:

$$\frac{7\pi}{12} = \frac{7\cancel{\pi}}{12}\,(\cancel{\text{radian}}) \cdot \frac{180^\circ}{\cancel{\pi}\,(\cancel{\text{radian}})} = \frac{7(180^\circ)}{12} = 105^\circ$$

Then write $105^\circ = 60^\circ + 45^\circ$

Or leave in radian measure:

Note:

$\frac{\pi}{6} = \frac{2\pi}{12}$

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

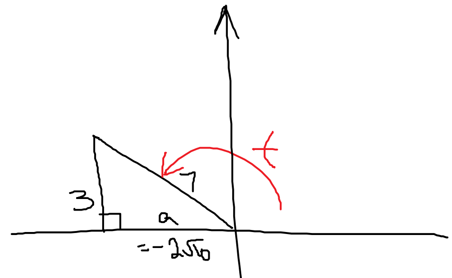
$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

Ex: Suppose $\cos(s) = -\frac{1}{7}$ and $\sin(t) = \frac{3}{7}$. Also suppose that s and t are both in Quadrant 2. Determine $\tan(s+t)$ and $\sin(s-t)$.



$1^2 + b^2 = 7^2$

$b^2 = 49 - 1$

$b^2 = 48$

$b = \pm\sqrt{48}$

$b = +\sqrt{16 \cdot 3}$

$= +4\sqrt{3}$

$\sin(s) = \frac{4\sqrt{3}}{7}$

$\cos(s) = \frac{-1}{7}$

$\tan(s) = \frac{4\sqrt{3}}{-1}$

$= -4\sqrt{3}$

$\cos(t) = -\frac{2\sqrt{10}}{7}$

$\sin(t) = \frac{3}{7}$

$\tan(t) = \frac{3}{-2\sqrt{10}}$

$= -\frac{3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

$= -\frac{3\sqrt{10}}{2 \cdot 10}$

$= -\frac{3\sqrt{10}}{20}$

$a^2 + 3^2 = 7^2$

$a^2 + 9 = 49$

$a^2 = 40$

$a = \pm\sqrt{40}$

$a = -\sqrt{4 \cdot 10}$

$a = -2\sqrt{10}$

$$\sin(s-t) = \sin(s)\cos(t) - \cos(s)\sin(t)$$

$$= \frac{4\sqrt{3}}{7}\left(-\frac{2\sqrt{10}}{7}\right) - \left(-\frac{1}{7}\right)\left(\frac{3}{7}\right)$$

$$= -\frac{8\sqrt{30}}{49} + \frac{3}{49} = \boxed{\frac{-8\sqrt{30}+3}{49}}$$

$$\tan(s+t) = \frac{\tan(s)+\tan(t)}{1-\tan(s)\tan(t)}$$

$$= \frac{-4\sqrt{3} + \left(-\frac{3\sqrt{10}}{20}\right)}{1-\left(-4\sqrt{3}\right)\left(-\frac{3\sqrt{10}}{20}\right)} = \frac{-4\sqrt{3} - \frac{3\sqrt{10}}{20}}{1 - \frac{12\sqrt{30}}{20}} \cdot \frac{20}{20}$$

$$= \boxed{\frac{-80\sqrt{3} - 3\sqrt{10}}{20 - 12\sqrt{30}}}$$