Double-Angle Identities

Know these:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

no need to memorize

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

More:

Variations on double-angle identity for cosine:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

($\cos^2\theta = 1-\sin^2\theta$)

$$\cos 2\theta = 1-\sin^2\theta-\sin^2\theta$$

$$\cos 2\theta = 1-2\sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

($\sin^2\theta = 1-\cos^2\theta$)

$$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1$$

3 variations:

Know the 1st one

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1-2\sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

Ex: Prove $\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A =$

$= \cos(A+A) = \cos A\cos A - \sin A\sin A$

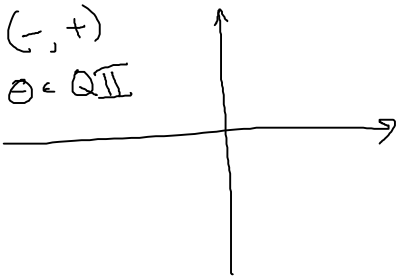
$= \cos^2 A - \sin^2 A$ ∎

Ex: Prove $\sin 2A = 2\sin A \cos A$

$$\sin 2A = \sin(A+A)$$
$$= \sin A\cos A + \cos A \sin A$$
$$= 2\sin A \cos A \quad \blacksquare$$

Ex: Find $\cos 2\theta$ and $\sin 2\theta$, given that $\cos\theta = -\frac{12}{13}$ and $\sin\theta > 0$.

$(-, +)$
$\theta \in$ QII



$$\cos^2\theta + \sin^2\theta = 1$$
$$\left(-\frac{12}{13}\right)^2 + \sin^2\theta = 1$$
$$\frac{144}{169} + \sin^2\theta = 1$$
$$\sin^2\theta = 1 - \frac{144}{169}$$
$$\sin^2\theta = \frac{169}{169} - \frac{144}{169}$$
$$\sin^2\theta = \frac{25}{169}$$
$$\sin\theta = \pm\sqrt{\frac{25}{169}}$$
$$\sin\theta = \pm\frac{5}{13}$$

QII, given info $\Rightarrow \sin\theta = +\frac{5}{13}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$$

$$\boxed{\sin 2\theta = -\frac{120}{169}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$
$$= \frac{144}{169} - \frac{25}{169}$$

$$\boxed{\cos 2\theta = \frac{119}{169}}$$