6.1

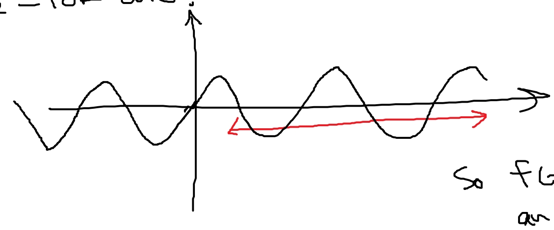
Monday, November 18, 2019 12:43 PM

6.1: Inverse Trigonometric Functions (Inverse Circular Functions)

Recall: Horizontal Line Test: Tells us if a function is one-to-one (has an inverse)

If any horizontal line intersects the graph more than once, the function is not 1-1.

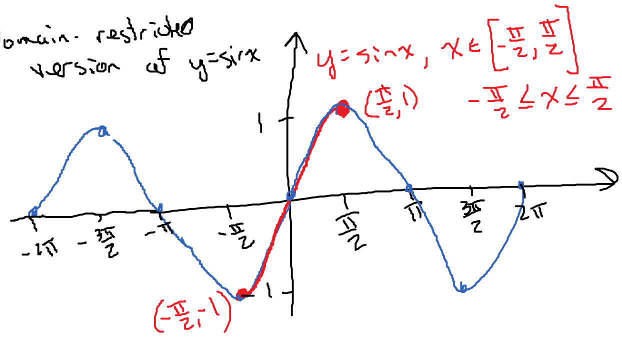
Is $f(x) = \sin(x)$ one-to-one? No!



So $f(x) = \sin x$ does not have an inverse.

To create a "pseudo-inverse" function, we restrict the domain.

Domain-restricted version of $y=\sin x$



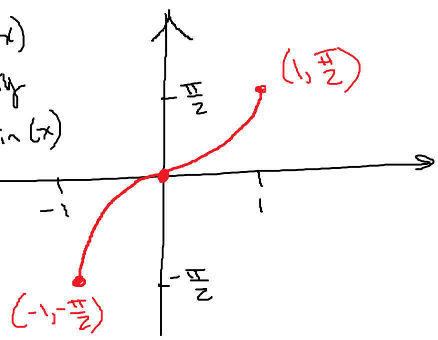
Shorthand:

$\in$ means "is an element of"

$y = \sin^{-1}(x)$

OR, equivalently

$y = \arcsin(x)$



Inverse Sine Function (or Arcsine Function)

$y = \sin^{-1}(x)$ or $y = \arcsin(x)$ means that $x = \sin y$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

In other words, $y = \sin^{-1}(x)$ is the number in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x.

Ex. Evaluate.

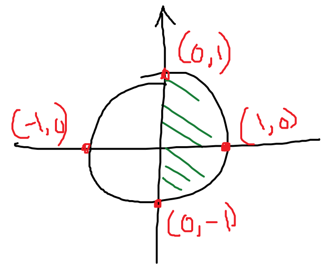
a) $\sin^{-1}(0) =$ ____

Let $\theta = \sin^{-1}(0)$

Then $\sin\theta = 0$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

So, what angle is θ? $\theta = 0$.

So, $\boxed{\sin^{-1}(0) = 0}$



b) $\sin^{-1}(-1) =$ ____

Let $\theta = \sin^{-1}(-1)$.

Then $\sin\theta = -1$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta = -\frac{\pi}{2}$

$\boxed{\sin^{-1}(-1) = -\frac{\pi}{2}}$

c) True or False?

$\frac{7\pi}{6} = -\frac{5\pi}{6}$

≈ 3.665 ≈ -2.618

False!

Note: $\frac{7\pi}{6}$ and $-\frac{5\pi}{6}$ are not the same number nor the same angle.

They are coterminal angles. The terminal sides of both are in the same location on unit circle, so all their trig functions are equal.

True or False?

$\frac{3\pi}{2} = -\frac{\pi}{2}$ False!

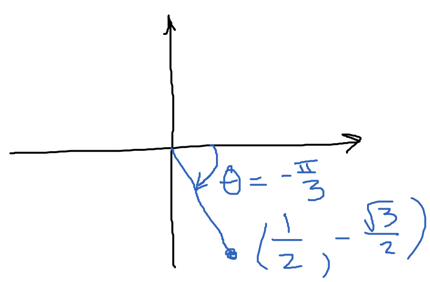
d) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) =$ ____

Let $\theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Then $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta = -\frac{\pi}{3}$

$\boxed{\text{So } \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}}$



e) $\arcsin\left(-\frac{5}{3}\right) =$ ______

Let $\theta = \arcsin\left(-\frac{5}{3}\right)$.

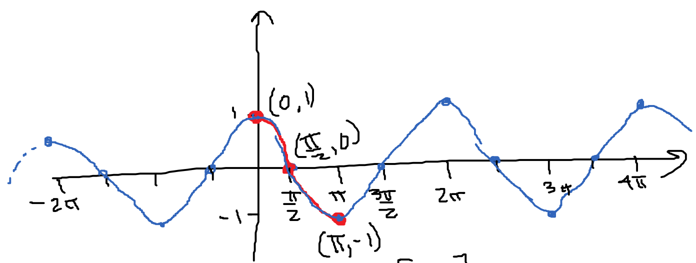
Then $\sin\theta = -\frac{5}{3}$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\theta = -1\frac{2}{3}$

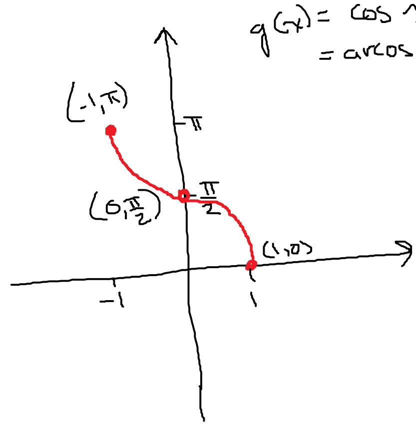
Impossible!! $\sin\theta$ and $\cos\theta$ are always between -1 and 1.

The Inverse Cosine (Arccosine) Function

$f(x) = \cos x$



$g(x) = \cos^{-1} x$
$= \arccos x$



Restrict the domain to $[0,\pi]$

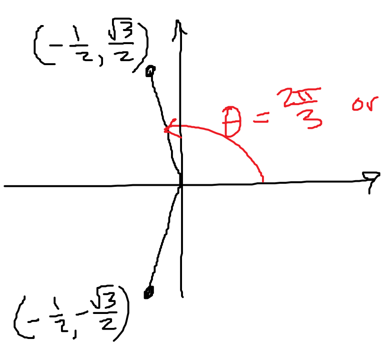
$$\boxed{y = \cos^{-1}x \quad \text{is equivalent to if and only if} \quad \cos y = x \quad \underline{\text{and}} \quad y \in [0,\pi] \quad 0 \le y \le \pi}$$

__Ex:__ Evaluate $\cos^{-1}\left(-\frac{1}{2}\right)$.

$\cos^{-1}\left(-\frac{1}{2}\right) =$ ______

Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$.

then $\cos\theta = -\frac{1}{2}$ <u>and</u> $\theta \in [0,\pi]$



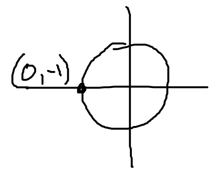
$\theta = \frac{2\pi}{3}$ or $\theta = 120^\circ$ (but we need the real-number value, so would switch to radian measure)

So $\boxed{\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}}$

Ex: $\arccos(-1) = \underline{\pi}$

Let $\theta = \arccos(-1)$

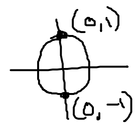
then $\cos\theta = -1$ and $\theta \in [0, \pi]$



Ex. $\arccos(0) = \underline{\frac{\pi}{2}}$

Let $\theta = \arccos(0)$

then $\cos\theta = 0$ and $\theta \in [0, \pi]$



Ex: $\cos^{-1}\left(\frac{5\pi}{6}\right) = \underline{\qquad}$

Let $\theta = \cos^{-1}\left(\frac{5\pi}{6}\right)$ then $\cos\theta = \frac{5\pi}{6}$ $\Rightarrow$ $\cos\theta \approx \frac{5(3)}{6} = \frac{15}{6} = \frac{5}{2} = 2\frac{1}{2}$

and $\theta \in [0, \pi]$

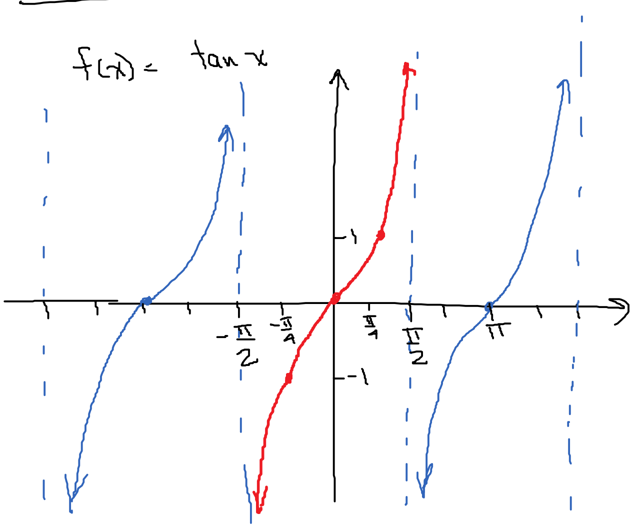
Not possible!

$-1 \leq \cos\theta \leq 1$

So $\boxed{\cos^{-1}\left(\frac{5\pi}{6}\right) \text{ is not defined.}}$

Inverse Tangent (arctangent) Function

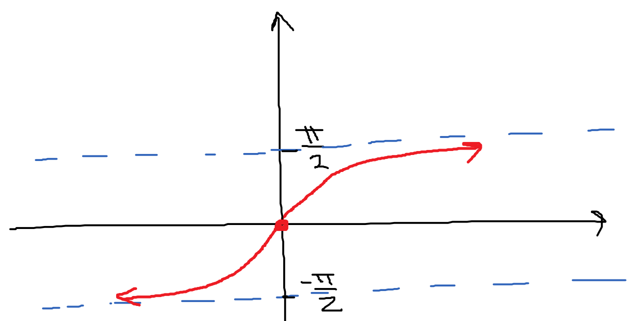
$f(x) = \tan x$



Restrict the domain to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$g(x) = \tan^{-1}(x)$

$= \arctan(x)$

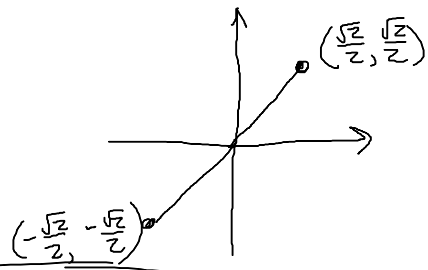


Know this graph!

Ex: $\arctan(1) = \frac{\pi}{4}$

Let $\theta = \arctan 1$

Then $\tan\theta = 1$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$\boxed{\tan^{-1}(x) = y \text{ if and only if } \tan y = x \text{ and } y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

Domain of $f(x) = \cos^{-1}(x)$ and $g(x) = \sin^{-1}x$: $[-1, 1]$

Domain of $h(x) = \tan^{-1}(x)$ is $(-\infty, \infty)$

There are also pseudo-inverse functions for cot, sec, csc.

skip those!