6.2: Solving Trig Equations, Part I

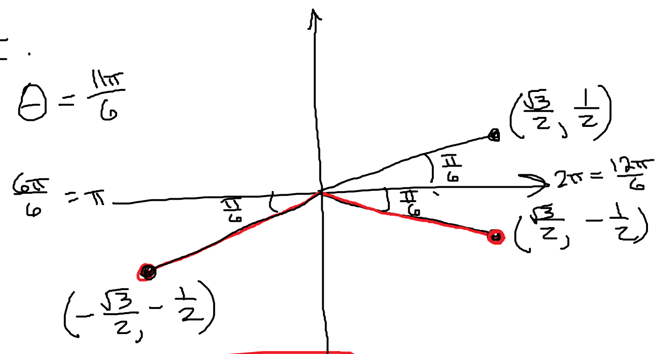
(we'll skip, at least for now, 6.3: Trig Eqns Part II)

Ex: Solve the equation on (a) $[0, 2\pi]$
(b) $(-\infty, \infty)$.

(a) solve on $[0, 2\pi]$ first:

$$\sin\theta = -\frac{1}{2}.$$

$$\theta = \frac{7\pi}{6}, \quad \theta = \frac{11\pi}{6}$$



we want Q3 and Q4.

Note: $\theta = -\frac{\pi}{6}$ also makes the eqn true but $-\frac{\pi}{6} \notin [0, 2\pi]$

(a) Solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

(b) Add integer multiples of 2π to $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ to get all real number solutions.

Solution Set on $(-\infty, \infty)$: $\boxed{\frac{7\pi}{6} + 2\pi k, \ \frac{11\pi}{6} + 2\pi k, \text{ where } k \text{ is any integer}}$

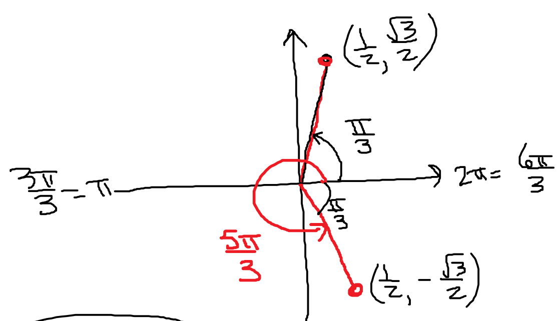
Ex: Solve on (a) $[0, 2\pi]$ and (b) $(-\infty, \infty)$.

$$2\cos x + 3 = 4$$
$$\quad -3 \quad -3$$

Isolate the trig function:

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$



(a) solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}}$

(b) Solution Set on $(-\infty, \infty)$: $\boxed{\left\{\frac{\pi}{3} + 2\pi k, \ \frac{5\pi}{3} + 2\pi k, \ k \text{ any integer}\right\}}$

Ex: Solve on (a) $[0, 2\pi]$, (b) all real numbers

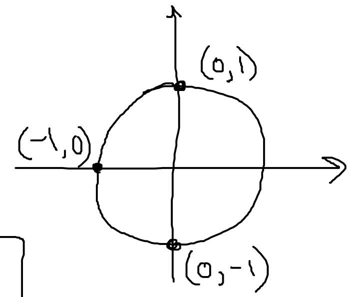
$\cos^2 x + \cos x = 0$

$\cos x(\cos x + 1) = 0$

$\cos x = 0$ OR $\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\cos x = -1$

$x = \pi$



(a) Solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \pi\right\}}$

(b) Sol'n Set on $(-\infty, \infty)$:

$\frac{\pi}{2} + 2k\pi$, $\frac{3\pi}{2} + 2k\pi$, $\pi + 2k\pi$,

where k is any integer.

We can combine $\frac{\pi}{2} + 2k\pi$ and $\frac{3\pi}{2} + 2k\pi$

into $\frac{\pi}{2} + k\pi$

Sol'n Set: $\boxed{\left\{\frac{\pi}{2} + k\pi, \ \pi + 2k\pi, \ k \text{ any integer}\right\}}$

OR $\boxed{\left\{\frac{\pi}{2} + k\pi, \ (2k+1)\pi, \ k \in \mathbb{Z}\right\}}$

Shorthand:

$\mathbb{Z}$ is the set of all integers.

$\mathbb{R}$ is the set of all real numbers

Notice: if k is an integer, then

$2k$ is even

$2k+1$ is odd

So $(2k+1)\pi$ means "odd multiples of π"

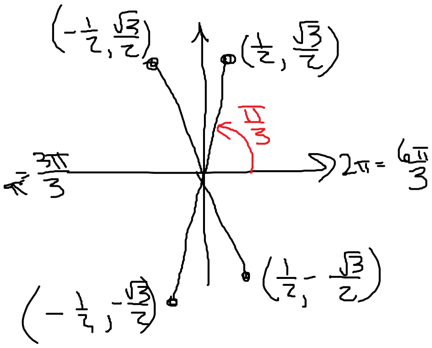
(so $\pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, \ldots$)

Ex: Solve $\tan^2 x = 3$ on $[0, 2\pi]$

$(\tan x)^2 = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \pm\frac{\sqrt{3}}{1} = \pm\frac{\sqrt{3}/2}{1/2} = \frac{\sin x}{\cos x}$



$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\boxed{\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$

All real number solutions: $\boxed{\left\{\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi, k \in \mathbb{Z}\right\}}$

Ex: Solve. $\cos^2\theta = \sin^2\theta + 1$

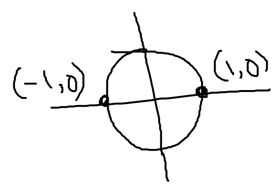
$1 - \sin^2\theta = \sin^2\theta + 1 \quad \Longleftarrow \quad \cos^2\theta = 1 - \sin^2\theta$

Recall:
$\cos^2\theta = 1 - \sin^2\theta$
$\sin^2\theta = 1 - \cos^2\theta$

$1 = 2\sin^2\theta + 1$
$-1 \qquad -1$

$0 = 2\sin^2\theta$

$\frac{0}{2} = \frac{2\sin^2\theta}{2}$



$0 = \sin^2\theta$

$\sin\theta = 0$

$\theta = 0°, 180°$ (on $[0°, 360°)$)

$\theta = 0, \pi$ (on $[0, 2\pi)$)

All degree solutions: $0° + 360°k$, $180° + 360°k$

consolidate into: $\boxed{0° + 180°k, \text{ k any integer}}$ or $\boxed{180°k, \text{ k any integer}}$

All radian solutions: $0 + 2\pi k$, $\pi + 2\pi k$

consolidate: $0 + \pi k$

so $\boxed{\pi k, \text{ k any integer}}$

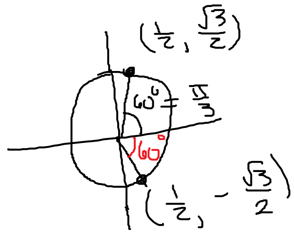
6.2.4

Ex. Solve.

$\sec\theta = 2$

$\frac{1}{\cos\theta} = 2$

$\cos\theta = \frac{1}{2}$



Find all solutions on $[0^\circ, 360^\circ)$:

$\boxed{\theta = 60^\circ, 300^\circ}$

Find all degree solutions:

$\boxed{60^\circ + 360^\circ k,\ 300^\circ + 360^\circ k,\ k \text{ any integer}}$

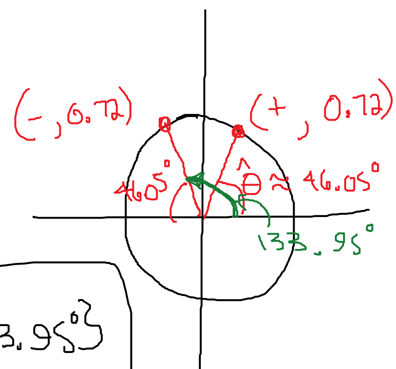
Approximate Solutions Using a Calculator

Ex. Solve on (a) $[0^\circ, 360^\circ)$ (b) all degree solutions.

$\sin x = 0.72$

1st find reference angle $\hat{\theta}$:

$\hat{\theta} \approx \sin^{-1}(0.72) \approx 46.05^\circ$



QII angle: $x = 180^\circ - 46.05^\circ$

(a) Solutions on $[0^\circ, 360^\circ)$: $\boxed{\{46.05^\circ, 133.95^\circ\}}$

(b) All degree solutions:

$\boxed{\{46.05^\circ + 360^\circ k,\ 133.95^\circ + 360^\circ k,\ k \text{ any integer}\}}$

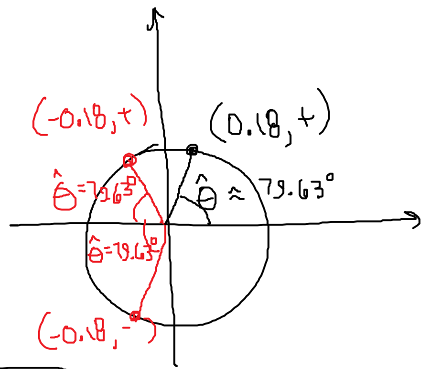
Ex: Solve on (a) $[0^\circ, 360^\circ)$, (b) all degree solutions.

$$\cos x + 0.18 = 0$$

Isolate trig function: $\cos x = -0.18$

Use inverse trig function to find reference angle $\hat{\theta}$: $\cos^{-1}(0.18) = \hat{\theta}$

calculator $\Rightarrow$ $\hat{\theta} = \cos^{-1}(0.18) \approx 79.63^\circ$



$\cos x$ will be negative in QII and QIII.

QII: $x = 180^\circ - 79.63^\circ = 100.37^\circ$

QIII: $x = 180^\circ + 79.63^\circ = 259.63^\circ$

(a) solution set on $[0^\circ, 360^\circ)$: $\boxed{\{100.37^\circ, 259.63^\circ\}}$

(b) all degree solutions: $\boxed{\{100.37^\circ + 360^\circ k,\ 259.63^\circ + 360^\circ k,\ k \text{ any integer}\}}$

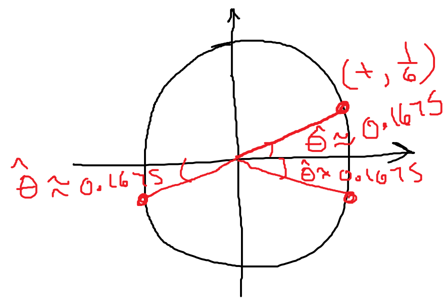
Ex: Solve on (a) $[0, 2\pi)$ and (b) $(-\infty, \infty)$.

$$\csc\theta = -6.$$

$$\sin\theta = -\frac{1}{6}$$

Find reference angle: $\sin^{-1}\left(\frac{1}{6}\right) = \hat{\theta}$

$$\hat{\theta} \approx 0.1675$$



$\sin\theta$ is neg, so I want QIII and QIV.

Q3: $\theta = \pi + 0.1675 \approx 3.309$

Q4: $\theta = 2\pi - 0.1675 \approx 6.116$

(a) Solution set on $[0, 2\pi)$: $\boxed{\{3.309, 6.116\}}$

(b) Solution set on $(-\infty, \infty)$: $\boxed{\{3.309 + 2k\pi,\ 6.116 + 2k\pi,\}}$ k any integer

Ex: Solve

$$\sin(5\theta) = 0.3$$

$$\hat{\theta} = \sin^{-1}(0.3) \approx 17.46^\circ$$

Q1, Q2,: Q1: $5\theta = 17.46^\circ$, $\theta = \frac{17.46^\circ}{5}$, Q2: $5\theta = 180^\circ - 17.46^\circ \approx 162.54^\circ$, $\theta \approx \frac{162.54^\circ}{5}$

then work out how many angles θ are in $[0^\circ, 360^\circ)$