6.2: Solving Trig Equations, Part I

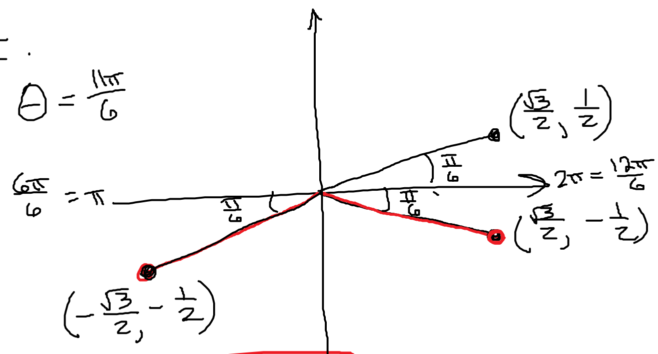
(we'll skip, at least for now, 6.3: Trig Eqns Part II)

Ex: Solve the equation on (a) $[0, 2\pi]$
(b) $(-\infty, \infty)$.

(a) solve on $[0, 2\pi]$ first:

$$\sin\theta = -\frac{1}{2}.$$

$$\theta = \frac{7\pi}{6},\quad \theta = \frac{11\pi}{6}$$



we want Q3 and Q4.

Note: $\theta = -\frac{\pi}{6}$ also makes the eqn true but $-\frac{\pi}{6} \notin [0, 2\pi]$

(a) Solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

(b) Add integer multiples of 2π to $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ to get all real number solutions.

Solution Set on $(-\infty, \infty)$: $\boxed{\frac{7\pi}{6} + 2\pi k,\ \frac{11\pi}{6} + 2\pi k, \text{ where } k \text{ is any integer}}$

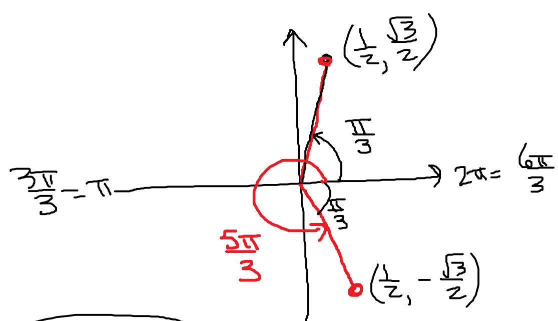
Ex: Solve on (a) $[0, 2\pi]$ and (b) $(-\infty, \infty)$.

$$2\cos x + 3 = 4$$
$$-3 \qquad -3$$

Isolate the trig function:

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$



(a) solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}}$

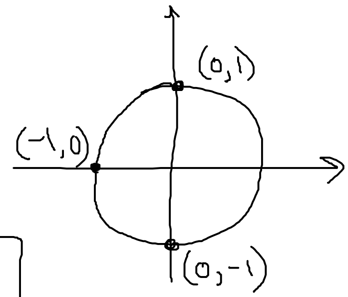
(b) Solution Set on $(-\infty, \infty)$: $\boxed{\left\{\frac{\pi}{3} + 2\pi k,\ \frac{5\pi}{3} + 2\pi k,\ k \text{ any integer}\right\}}$

Ex: Solve on (a) $[0, 2\pi]$, (b) all real numbers

$$\cos^2 x + \cos x = 0$$
$$\cos x(\cos x + 1) = 0$$

$\cos x = 0$ OR $\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\cos x = -1$, $x = \pi$



(a) Solution Set on $[0, 2\pi]$: $\boxed{\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \pi\right\}}$

(b) Sol'n Set on $(-\infty, \infty)$:

$\frac{\pi}{2} + 2k\pi$, $\frac{3\pi}{2} + 2k\pi$, $\pi + 2k\pi$,

where k is any integer.

We can combine $\frac{\pi}{2} + 2k\pi$ and $\frac{3\pi}{2} + 2k\pi$

into $\frac{\pi}{2} + k\pi$

Sol'n Set: $\boxed{\left\{\frac{\pi}{2} + k\pi, \ \pi + 2k\pi, \ k \text{ any integer}\right\}}$

OR $\boxed{\left\{\frac{\pi}{2} + k\pi, \ (2k+1)\pi, \ k \in \mathbb{Z}\right\}}$

Shorthand:

$\mathbb{Z}$ is the set of all integers.

$\mathbb{R}$ is the set of all real numbers

Notice: if k is an integer, then

$2k$ is even

$2k+1$ is odd

So $(2k+1)\pi$ means "odd multiples of π"

(so $\pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, \ldots$)

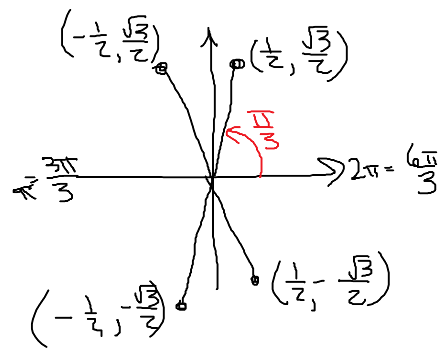
Solve

Ex: $\tan^2 x = 3$ on $[0, 2\pi]$

$(\tan x)^2 = 3$

$\tan x = \pm\sqrt{3}$

$\tan x = \pm\frac{\sqrt{3}}{1} = \pm\frac{\sqrt{3}/2}{1/2} = \frac{\sin x}{\cos x}$



$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\boxed{\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$

All real number solutions: $\boxed{\left\{\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi, k \in \mathbb{Z}\right\}}$