1342-Notes_Navidi_6-1_discrete-random-variables

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6.1: Random Variables

A *random variable* is a quantitative variable that represents the outcomes of a probability experiment. Thus, the value of a random variable depends on chance.

A *discrete random variable* is a random variable that takes on a finite or countably infinite number of values.

A *continuous random variable* is a random variable that takes on all values on an interval of the real number line (i.e., it is not countable).

A *discrete probability distribution* is a function that assigns a probability to each outcome. (So, it assigns a probability to each value of the discrete random variable). If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive. The probability distribution can be described by a table, graph, or mathematical formula.

Notation:

If X is a random variable, then the probability of X taking on the value x is denoted $P(X = x)$. For example, the probability of X taking on the value 3 is $P(X = 3)$. The probability of X taking on a values of at least 5 is denoted $P(X \geq 5)$.

Example 1: A probability distribution is given by the table below.

x	12	13	14	15	16	17	18
$P(X = x)$	0.32	0.18	0.13	0.11	0.10	0.08	0.08

Note: $\sum P(x) \cdot x = 1$

a) What is $P(x = 17)$?

$P(X = 17) = \boxed{0.08}$

b) What is $P(x \geq 16)$?

$P(X \geq 16) = P(X=16) + P(X=17) + P(X=18) = 0.10 + 0.08 + 0.08 = \boxed{0.26}$

c) What is $P(x > 13)$?

$P(x > 13) = P(x=14) + \ldots + P(x=18)$

or use the complement:

E^c: $X \leq 13$

$P(E^c) = P(X=12) + P(X=13)$

$= 0.32 + 0.18 = 0.5$

$P(E) = P(x > 13) = 1 - 0.5 = \boxed{0.5}$

Example 2: A car repair shop's records show that 25 clients have 6 cars, 83 clients have 5 cars, 140 clients have 4 cars, 183 clients have 3 cars, and 209 clients have 2 cars. The remaining 313 clients own only 1 car. Determine the probability distribution for the number of cars owned by the shop's clients. Construct a probability histogram. If the manager decides to randomly call a customer and invite him or her to complete a satisfaction survey, what is the probability that the customer called has 2 or fewer cars?

$n = \Sigma$ frequencies

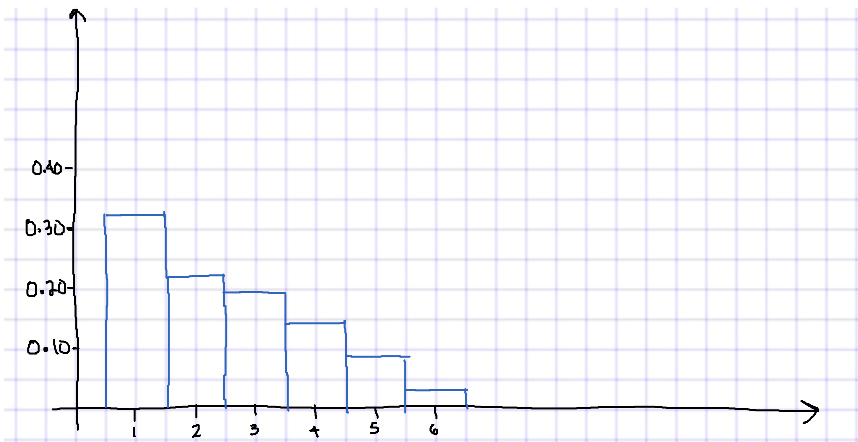
$= 25 + 83 + 140 + 183 + 209 + 313$

$= 953$

X = number of cars owned by a customer

x	6	5	4	3	2	1
$P(X=x)$	25/953	83/953	140/953	183/953	209/953	313/953
	≈ 0.0262	0.0871	0.1469	0.1920	0.2193	0.3284

Check: $0.0262 + 0.0871 + \ldots + 0.3284 = 0.9999$ (close enough to be rounding error)



$P(X \le 2) = P(X=2) + P(X=1)$

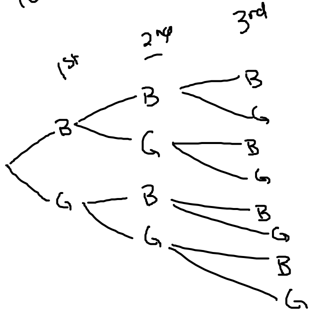
$= 0.2193 + 0.3284$

$= \boxed{0.5477}$ This is the prob. that the customer who was called has 2 or fewer cars.

Example 3: Create a probability distribution to represent the number of girls in a three-child family. Assume that boys and girls are equally likely. Construct the probability histogram. What is the probability that a three-child family has exactly one girl? What is the probability that a three-child family has at least one girl?

$S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$

(outcomes that can happen in a 3-child family)



$n(S) = 8$

B = boy
G = Girl

Multiplication Principles:

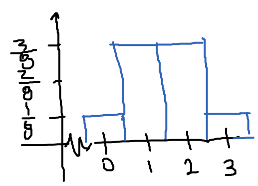
There are $2 \cdot 2 \cdot 2 = 8$ possibilities

This is an equally likely sample space.

Let X = number of girls

$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$
$= \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \boxed{\frac{7}{8}}$

x	$P(X=x)$
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$



Mean of a discrete random variable:

The Mean (Expected value) of a Discrete Random Variable:

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$

Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X.

For the 3-child family, the expected value of the number of girls is

$\mu = E(X) = 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right) = \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \boxed{1\frac{1}{2}}$

Example 4: A probability distribution is given by the table below. Find the mean (the expected value of *X*).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

$\mu = E(X) = 3(0.15) + 4(0.2) + 5(0.30) + 6(0.12) + 7(0.08) + 8(0.10) + 9(0.05)$

$= \boxed{5.28}$

Example 5: Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for a gift basket worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = net winnings for 1 ticket bought

Outcomes for 1 ticket	X	$P(X)$
Gift basket	$200 − $1 = $199	$\frac{1}{1000} = 0.001$
Restaurant gift certificate	$50 − $1 = $49	$\frac{3}{1000} = 0.003$
Win nothing	−$1	$\frac{996}{1000} = 0.996$

Check: Do probabilities add up to 1? Yes

$E(X) = \mu$

$= \$199(0.001) + \$49(0.003) - \$1(0.996)$

$= \boxed{-\$0.65}$ expected net winnings for 1 ticket

Example 6: Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

X = value of policy to customer

	X	$P(X)$
Big wreck	$100 000 − $2300 = $97 700	
Little wreck	$30 000 − $2300 = $27 700	
No wreck	−$2300	

Standard deviation of a discrete random variable:

<u>The Standard Deviation of a Discrete Random Variable</u>**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\sigma = \sqrt{(x_1-\mu)^2 p_1 + (x_2-\mu)^2 p_2 + ... + (x_n-\mu)^2 p_n}$$
$$= \sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p_i}.$$

<u>Example 7:</u> Calculate the mean and standard deviation of the probability distribution.

x	$P(X=x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

$\mu = E(X) = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04) = \boxed{1.64} = \mu$

x	$(x-\mu)^2 p$
0	$(0-1.64)^2(0.11) \approx 0.295856$
1	$(1-1.64)^2(0.32) \approx 0.131072$
2	$(2-1.64)^2(0.43) \approx 0.055728$
3	$(3-1.64)^2(0.10) \approx 0.18496$
4	$(4-1.64)^2(0.04) \approx 0.222784$

$\Sigma(x-\mu)^2 p = 0.8904 = \sigma^2$

Std Dev. $\sigma = \sqrt{\sigma^2} \approx \sqrt{0.8904} \approx 0.9436$

Variance of X: $\sigma^2 \approx 0.8904$

Std deviation of X: $\sigma \approx 0.9436$

Example 8: Use the frequencies to construct a probability distribution for the random variable *X*, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of *X*.

Number of Games	Frequency
1	37
2	45
3	29
4	12
5	4