6.1: Random Variables

A *random variable* is a quantitative variable that represents the outcomes of a probability experiment. Thus, the value of a random variable depends on chance.

A *discrete random variable* is a random variable that takes on a finite or countably infinite number of values.

A *continuous random variable* is a random variable that takes on all values on an interval of the real number line (i.e., it is not countable).

A *discrete probability distribution* is a function that assigns a probability to each outcome. (So, it assigns a probability to each value of the discrete random variable). If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive. The probability distribution can be described by a table, graph, or mathematical formula.

Notation:

If X is a random variable, then the probability of X taking on the value x is denoted $P(X = x)$. For example, the probability of X taking on the value 3 is $P(X = 3)$. The probability of X taking on a values of at least 5 is denoted $P(X \geq 5)$.

Example 1: A probability distribution is given by the table below.

x	12	13	14	15	16	17	18
$P(X = x)$	0.32	0.18	0.13	0.11	0.10	0.08	0.08

a) What is $P(x = 17)$?

b) What is $P(x \geq 16)$?

c) What is $P(x > 13)$?

<u>Example 2:</u> A car repair shop's records show that 25 clients have 6 cars, 83 clients have 5 cars, 140 clients have 4 cars, 183 clients have 3 cars, and 209 clients have 2 cars. The remaining 313 clients own only 1 car. Determine the probability distribution for the number of cars owned by the shop's clients. Construct a probability histogram. If the manager decides to randomly call a customer and invite him or her to complete a satisfaction survey, what is the probability that the customer called has 2 or fewer cars?

Example 3: Create a probability distribution to represent the number of girls in a three-child family. Assume that boys and girls are equally likely. Construct the probability histogram. What is the probability that a three-child family has exactly one girl? What is the probability that a three-child family has at least one girl?

Mean of a discrete random variable:

The Mean (Expected value) of a Discrete Random Variable**:**

Suppose that a random variable *X* can take on the *n* values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of *X* is

$$\mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n .$$

Suppose an experiment is repeated many times, and the values of *X* are recorded and then averaged. As the number of repetitions increases, the average value of *X* will become closer and closer to μ. For that reason, the mean is called the *expected value* of *X*.

Example 4: A probability distribution is given by the table below. Find the mean (the expected value of X).

x	3	4	5	6	7	8	9
$P(X = x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

Example 5: Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for a gift basket worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

Example 6: Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company?

Standard deviation of a discrete random variable:

<u>The Standard Deviation of a Discrete Random Variable</u>**:**

Suppose that a random variable *X* can take on the *n* values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\sigma = \sqrt{(x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + ... + (x_n - \mu)^2 p_n}$$
$$= \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i} \quad .$$

<u>Example 7:</u> Calculate the mean and standard deviation of the probability distribution.

x	$P(X = x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

Example 8: Use the frequencies to construct a probability distribution for the random variable *X*, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of *X*.

Number of Games	Frequency
1	37
2	45
3	29
4	12
5	4