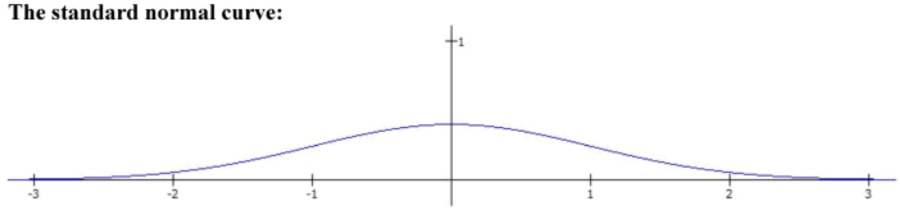
7.1: Areas Under the Standard Normal Curve

Areas under the standard normal curve:

One of the many remarkable things about normal curves is that for any normal curve, with any mean and standard deviation, the areas under the curve (and thus the associated probabilities and percentages) can be determined easily by using only one table.

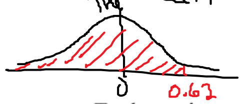
The standard normal curve:



Properties of the Standard Normal Curve:

1. It is bell-shaped and symmetric about the line $x = 0$.
2. The standard normal distribution has mean 0 and standard deviation 1. ($\mu = 0$, $\sigma = 1$)
3. The area between the curve and the horizontal axis is always 1. (This corresponds to the fact that all the probabilities in a distribution must add up to 1.)
4. The curve approaches the *x*-axis asymptotically. (It gets closer and closer to the *x*-axis but never intersects it; the curve extends indefinitely in both directions).
5. The values on the *x*-axis can be thought of as *z*-scores.
6. Regardless of the shape,
   - 68.3% of the area is between 1 and -1.
   - 95.4% of the area is between 2 and -2.
   - 99.7% of the area is between 3 and -3.
7. The curve has inflection points at 1 and -1. inflection point: where the concavity changes direction.

Ex $\frac{1}{2}$: Find the area under the standard normal curve to the left of 0.62



z	0.00	0.01	0.02
⋮			
0.6			0.7324

$\boxed{P(Z < 0.62) = 0.7324}$

Area left of 0.62 is 0.7324

To determine areas under the curve and thus probabilities, we'll use a table. ~~(See Table II in Appendix A,~~ or inside the cover of your book, or use the handout.) See Table A.2 7.1.2 in appendix

Example 1: Find the area under the standard normal curve between -1.94 and 0.62.

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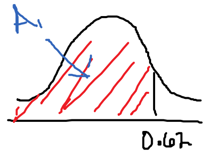

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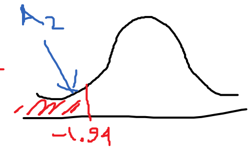


z	...	0.04
⋮		
−1.9		0.0262

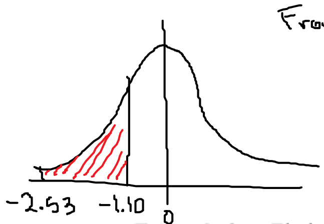
Area $= A_1 - A_2$
$= 0.7324 - 0.0262$
$= \boxed{0.7062}$

From table, $A_1 = 0.7324$, $A_2 = 0.0262$

Example 2: Find the area under the standard normal curve between -1.1 and -2.53.

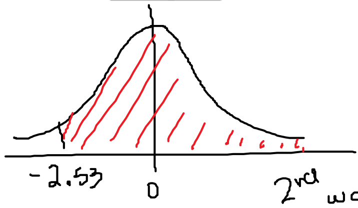
From table, Area left of $z = -2.53$ is $A_1 = 0.0057$

Area left of $z = -1.10$ is $A_2 = 0.1357$



Area $= 0.1357 - 0.0057$
$= \boxed{0.1300}$

Example 3: Find the area under the standard normal curve to the right of -2.53.



Two ways to do this from our table:

1st way) use the complement of the area left of -2.53.
This Area is $P(Z < -2.53) = 0.0057$
Area to the right is $P(Z > -2.53) = 1 - 0.0057$
$= \boxed{0.9943}$

2nd way) Look up $Z = 2.53$ instead; $P(Z < 2.53) = \boxed{0.9943}$ which is equal to the area we want because of symmetry

Example 4: Find the area under the standard normal curve to the left of -1.1.

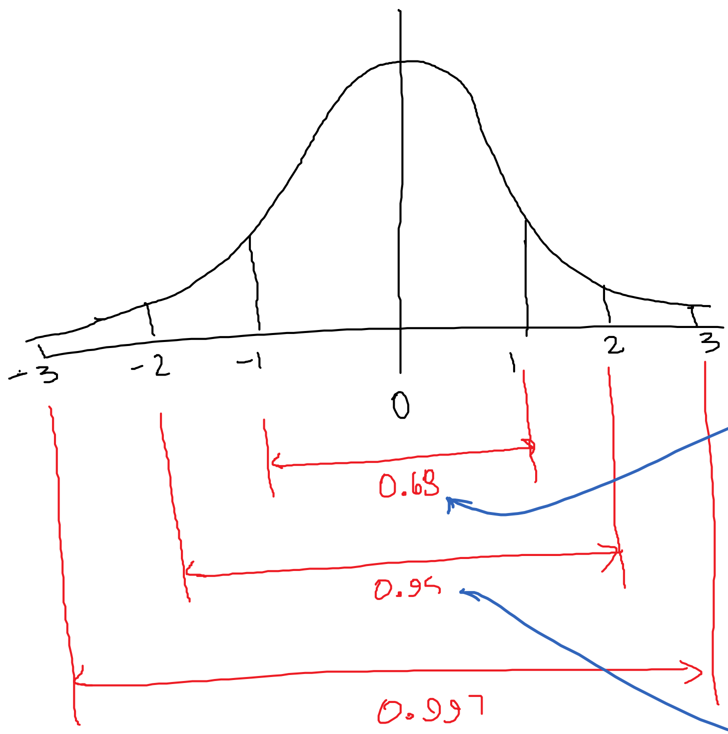
Example 5: Find the area under the standard normal curve to the left of 2.08.

Finding a z-score associated with a certain area:

The symbol z_α indicates the z-score which has an area α to its right.

<u>Example 6:</u> Find $z_{0.10}$, $z_{0.05}$, $z_{0.025}$, and $z_{0.0015}$.

Remember Empirical Rule For bell-shaped



Look up areas for

$z = 1.00$ and $z = -1.00$ in table

$P(Z < 1.00) = 0.8413$

$P(Z < -1.00) = 0.1587$

$P(-1 < Z < 1) = 0.8413 - 0.1587$

$= \boxed{0.6826}$

Look up areas for $Z = \pm 2$

$P(Z < 2) = 0.9772$

$P(Z < -2) = 0.0228$

$P(-2 < Z < 2) = 0.9772 - 0.0228$

$= \boxed{0.9544}$