Hypothesis Testing for one population mean, sigma known

9.2: ~~Critical-Value Approach to Hypothesis Testing~~

How do we decide whether the null hypothesis is tenable? Or whether there is evidence in favor of the alternative hypothesis? There are two approaches. In both approaches, we set the α-level and state the null and alternative hypotheses before the sample data is analyzed. Then:

Approach #1: *p*-value approach to hypothesis testing (we'll omit):

First, we calculate the test statistic. If we are interested in testing a hypothesis about the population mean, then the test statistic is ~~the sample mean~~. $\frac{\bar{x}-\mu_x}{\sigma_{\bar{x}}}$

Then we use the test statistic to calculate a *p*-value, often by using technology. The *p*-value, called the *observed significance level*, is the probability of obtaining a sample statistic at least as extreme as that observed in the sample, given that the null hypothesis is true.
(probability of being further out in the tails)

A result is said to be *statistically significant* if the *p*-value is less than the predetermined α level.
$\alpha = 0.05 \Rightarrow$ we reject H_0 is $p < 0.05$

If the *p*-value is less then or equal to α, we reject the null hypothesis.
If the *p*-value is greater than α, we fail to reject the null hypothesis.

Note: We cannot prove the null hypothesis is true. We never accept the null hypothesis. The closest we can come to accepting the null hypothesis is to conclude that there is not enough evidence to reject it.

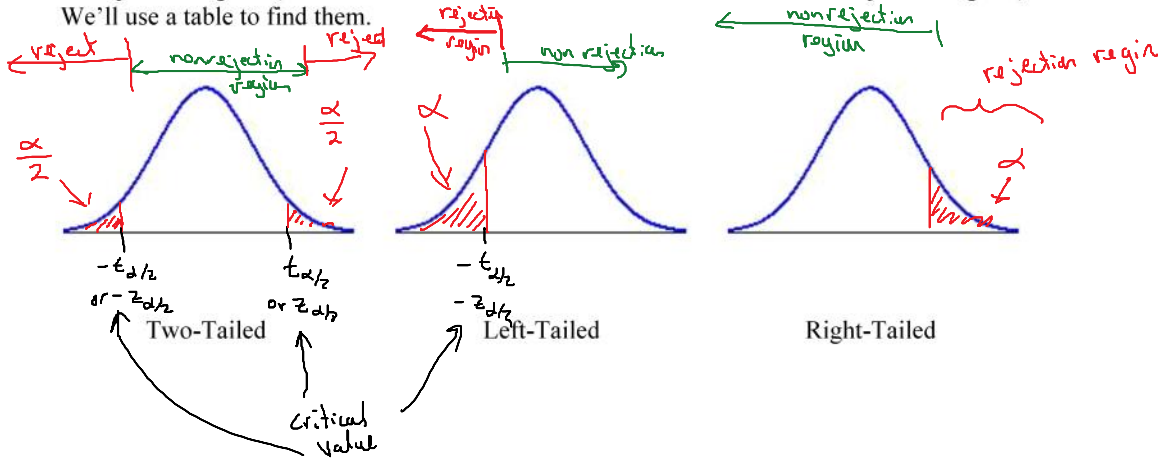
Approach #2: Critical-value approach to hypothesis testing (we'll use):

Determine the rejection region, nonrejection region, and critical value(s).

Rejection region: Set of values for the test statistic that lead to rejection of the null hypothesis.

Nonrejection region: Set of values for the test statistic that do not lead to rejection of the null hypothesis.

Critical value(s): Value(s) of the test statistic that separate the rejection region from the nonrejection region. (The critical values are considered to be included in the rejection region.) We'll use a table to find them.



Calculate the test statistic and compare it to the critical value.

- If the test statistic falls in the rejection region, reject the null hypothesis.
- If the test statistic falls in the nonrejection region, do not reject the null hypothesis.

Obtaining critical values for a one-mean *z*-test: $H_0: \mu = \mu_0$

We test a hypothesis about one population mean, in which the ~~null~~ alternative hypothesis $\cancel{H_0}$ H_1 is $\mu \neq \mu_0$, $\mu < \mu_0$, or $\mu > \mu_0$.

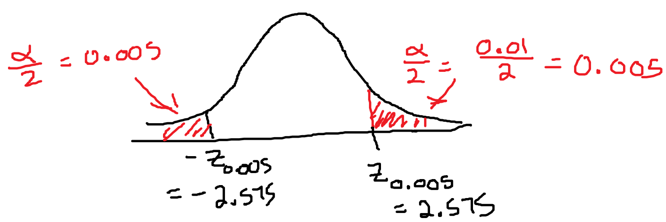
The one-mean *z*-test is used when the population standard deviation is known and the variable under consideration is normally distributed. The test statistic is

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}.$$

<u>Example 1:</u> Determine the critical value(s) for a two-tailed z-test with $\alpha = 0.01$. Sketch the rejection region.

$\alpha = 0.01$

$$\boxed{\begin{array}{l}\text{Critical values are:} \\ z_{0.005} = 2.575 \\ -z_{0.005} = -2.575\end{array}}$$



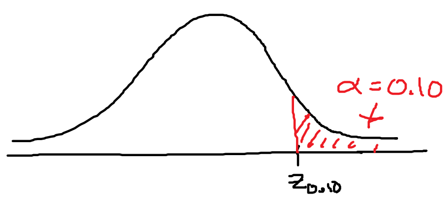
Area to the left of $z_{0.005}$ is $1 - 0.005 = 0.995$

Look up Area = 0.995 in z-table: $z_{0.005} = 2.575$

<u>Example 2:</u> Determine the critical value(s) for a right-tailed z-test with $\alpha = 0.10$. Sketch the rejection region.

$\alpha = 0.10$

$1 - 0.10 = 0.90$ = area to the left of $z_{0.10}$



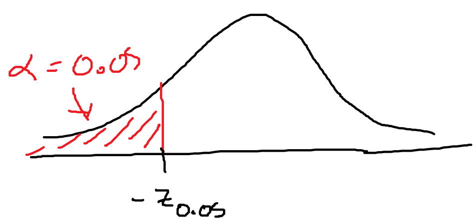
Look up area = 0.90 in z-table

Critical value: $z_{0.10} \approx 1.28$

<u>Example 3:</u> Determine the critical value(s) for a left-tailed z-test with $\alpha = 0.05$. Sketch the rejection region.

Look up Area = 0.05 in z-table.

$\Rightarrow -z_{0.05} = -1.645$



$$\boxed{\text{Critical value: } -z_{0.05} = -1.645}$$

Example 4: Suppose that a random sample of 40 fifth graders in a particular area were given a national standardized test. ~~It is known that the nationwide population mean on the test is 100 and the population standard deviation is 18.~~ The mean score for this sample of fifth graders was 92. Can it be concluded that this ~~group of students~~ year's 5th graders differed from the national average on their knowledge of the content area covered by the test? Use a significance level of 0.05.

$\sigma = 18$

this year's 5th graders

Historical records show that the national average is 100.
We can assume the current year's pop. std dev. is 18, the same as past years.

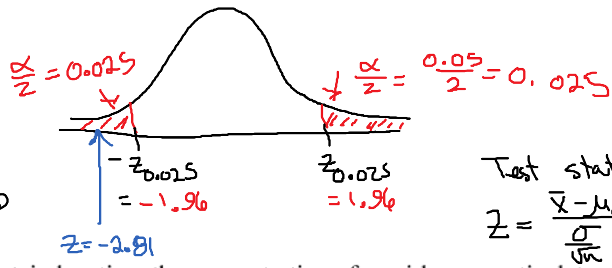
$n = 40$

$\bar{x} = 92$

$\alpha = 0.05$

$H_0: \mu = 100$

$H_1: \mu \neq 100$



std error: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{40}} \approx 2.846$

Look up area $= 0.025$

$\Rightarrow -z_{0.025} = -1.96$

Test statistic:

$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{92 - 100}{2.846} \approx -2.81$

$z = -2.81$ is in rejection region, so we $\boxed{\text{Reject } H_0}$

There is evidence that the 5th graders differ from 100

Example 5: At a certain location, the concentration of an airborne particulate was measured on 42 randomly chosen days of a calendar year. The mean of those measurements was 335 parts per million (ppm). Suppose it can be assumed that the standard deviation of these daily measurements, if they were taken over all calendar days, is 80 ppm. Suppose the EPA considers a level of 300 ppm or below to be safe. Can it be concluded that this area has a level of this pollutant that is higher than what is considered safe? Use a significance level of 0.01.