9.3: Hypothesis Tests for One Population Mean, Sigma Unknown

In practice, when we are using a sample to make inferences about the population mean, it is rare for us to know the population standard deviation.

Instead, we must use the sample standard deviation, s, as a point estimate of the population standard deviation, σ.

When using s as an estimate for σ, we cannot use a z-test, because $\dfrac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ is not normally distributed.

The *t*-test for one population mean:

When using s as an estimate for σ, we use the Student t-distribution.

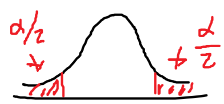
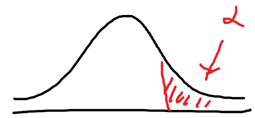
In order to use this procedure, we need to know (or be able to reasonably assume) that the variable of interest follows a normal distribution, or we must have a large sample size ($n \geq 30$).

In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\dfrac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

<u>Hypothesis Testing for a Population Mean:</u>

<u>Step 1</u>: Determine the significance level α .

<u>Step 2</u>: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
 Rejection Region	 Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

<u>Step 3</u>: Use your α level and hypotheses, sketch the rejection region.

<u>Step 4</u>: Compute the test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.

<u>Step 5</u>: Use a table (Table IV, on page A-13) to determine the <u>critical value for t</u> associated with your rejection region.
Recall: df = n-1

<u>Step 6</u>: Determine whether the value of t calculated from your sample (in Step 3) is in the rejection region.

- If t is in the rejection region, reject the null hypothesis.
- If t is not in the rejection region, do not reject the null hypothesis.

<u>Step 7</u>: State your conclusion.

Assumptions met? Need either 1) $n \geq 30$
Yes 2) temp normally distribution in population ✓

Example 1: The normal human body temperature is widely accepted to be 98.6° F and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans has a mean body temperature of 98.6° F. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of 98.3° F with a standard deviation of 0.7° F. Perform an appropriate hypothesis test at the 95% confidence level.

Hypotheses

$H_0: \mu = 98.6°F$

$H_1: \mu \neq 98.6°F$

$\alpha = 1 - 0.95 = 0.05$

Sample Information

$n = 20$ $df = n - 1 = 19$

$\bar{x} = 98.3°F$

$s = 0.7°F$

Conclusion

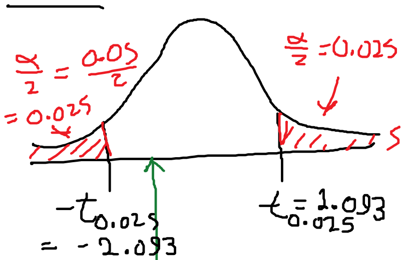
Do not reject H_0.
This sample does not provide evidence that the Alaskans' body temperature differs from 98.6°F

Find critical value

Use t-table, because we do not have pop. std dev σ

For $df = 19$, right tail $= 0.025$,
Critical value is $t_{0.025} = 2.093$

Picture



$t = -1.92$ not in rejection region

Calculate Test Statistic

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}} = \frac{98.3 - 98.6}{\frac{0.7}{\sqrt{20}}} = -1.92$$

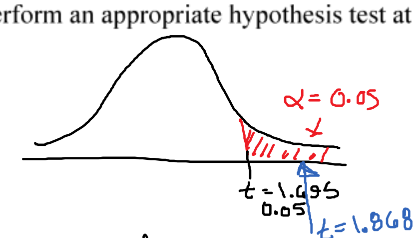
Std error: $\frac{s}{\sqrt{n}} = \frac{0.7}{\sqrt{20}} \approx 0.1565$

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level. → 1-tailed

$H_0: \mu = 2 mcg/dL$

$H_1: \mu > 2 mcg/dL$

$\alpha = 1 - 0.95 = 0.05$



Sample info:
$n = 35$
$\bar{x} = 2.60$
$s = 1.9$
σ is not known! So can't use z
$df = 35 - 1 = 34$

Find critical value: look up $df = 34$, right-tail area $= 0.05$ in t-table

Critical value: $t_{0.05} = 1.691$

std error: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$
$= \frac{1.9}{\sqrt{35}} \approx 0.3212$

Calculate test statistic for our sample:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2.60 - 2}{0.3212} \approx 1.868$$

This is in the rejection region, so we $\boxed{\text{Reject } H_0}$

$\boxed{\text{There is evidence from this sample that the kids' lead level are above 2mcg/dL.}}$

Ex 1 cont'd

Let's create a confidence interval for Example 1.

Sample info:

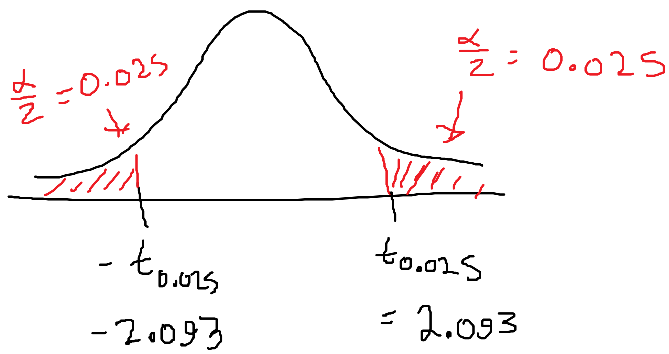
$n = 20$

$\bar{x} = 98.3^\circ F$

$s = 0.7^\circ F$

Create the 95% Confidence Interval.

Find the critical value of t: $2.093 = t_{0.025}$



df = 19

right-tail area 0.025

Calculate standard error: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{0.7}{\sqrt{20}} \approx 0.1565$

Confidence Interval: $\bar{x} \pm t_{0.025} \cdot \frac{s}{\sqrt{n}}$

$$\bar{x} \pm \left(\text{critical value}\right)\left(\text{std error}\right)$$

Upper bound: $98.3 + 2.093\,(0.1565) = 98.6276^\circ F \approx 98.63^\circ F$

Lower bound: $98.3 - 2.093\,(0.1565) = 97.9724^\circ F \approx 97.97^\circ F$

$$\boxed{95\%\ \text{CI is}\ (97.97^\circ F,\ 98.63^\circ F)}$$

Notice: Compare to hypothesis test we did earlier.

Our benchmark 98.6^0 is captured inside the 95% CI.

This corresponds to H_0 not being rejected at 95% confidence level

If the benchmark value fell outside of the CI, that means

H_0 is rejected (all this for 2-sided test only)

<u>Example 3:</u> Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

<u>Example 4:</u> Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analysist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.