9.4: Hypothesis Tests for One Population Proportion

In a hypothesis test for one population proportion, we are testing the hypothesis that the population proportion p is equal to a benchmark value p_0.

As long as $np \geq 10$ and $n(1-p) \geq 10$, we can assume that the sample proportion $\hat{p} = \frac{x}{n}$ is normally distributed. Therefore the critical values come from the *z*-distribution.

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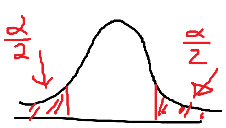
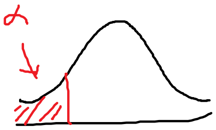
Hypothesis Testing for a Population Proportion:

Step 1: Determine the significance level α.

Step 2: Check that the assumptions are satisfied (simple random sample, $np \geq 10$ and $n(1-p) \geq 10$).

Step 3: Determine the null and alternative hypotheses.

p_0: benchmark value for the proportion – the number we are testing the sample against.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0: p = p_0$ $H_a: p \neq p_0$	$H_0: p = p_0$ $H_a: p < p_0$	$H_0: p = p_0$ $H_a: p > p_0$
$\frac{\alpha}{2}$ $\frac{\alpha}{2}$  Rejection Region	α  Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($p > p_0$ for a left-tailed test or $p < p_0$ for a right-tailed test) is not possible or is of zero interest.

Step 4: Use your α level and hypotheses, sketch the rejection region.

Step 5: Use a normal curve table (Table ____ , on page _____) to determine the critical value for z associated with your rejection region.

Step 6: Compute the test statistic $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.

$$\text{test statistic (z or t)} = \frac{\text{Subtract what we're comparing}}{\text{std. error}}$$

Step 7: Determine whether the value of z calculated from your sample (in Step 6) is in the rejection region.

- If z is in the rejection region, reject the null hypothesis.
- If z is not in the rejection region, do not reject the null hypothesis.

Step 8 State your conclusion.

one-tailed test

Example 1: A local police department claims that $\boxed{\text{over}}$ 70% of false alarms from home security systems are caused by human error (e.g., a house-sitter or child sets off the alarm by mistake). A random sample of 450 police files for false alarm calls are analyzed. Of the 450 false alarms in the sample, 324 were caused by human error. Does this sample provide evidence to support the police department's claim? Use a 95% confidence level.

Sample info

$n = 450$

sample proportion: $\hat{p} = \frac{324}{450} \approx 0.72$

Assumptions met?

Caused by Human Error	Not Human Error
324	126

Both ≥ 10, so can use normal

hypotheses

p = proportion of false alarms caused by human error

H_0: $p = 0.70$

H_1: $p > 0.70$

$\alpha = 0.05$

$p_0 = 0.7$ (benchmark)

$1 - p_0 = q_0 = 0.3$

Compute test statistic

Std error: use benchmark for p_0:

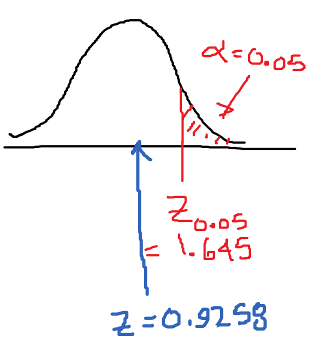
$$\sigma_{\hat{p}} \approx \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.7(0.3)}{450}}$$

≈ 0.0216025 (store in calculator)

Test statistic: $$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{0.72 - 0.70}{0.0216025}$$

≈ 0.9258

Picture



Find critical value

In normal table, look up Area = 0.95

Corresponds to $z_{0.05} = 1.645$

Conclusion

$z = 0.9258$, so not in the rejection region.

$\boxed{\text{Do not reject } H_0}$

This sample does not provide evidence that the proportion of false alarms caused by human error is over 70%.

(a statistically significant difference from 0.70 was not found)

Example 2: According to genetic theory, if two red/sorrel Appaloosa-spotted horses are mated, there is a 0.75 probability that the foal will be a red/sorrel Appaloosa (otherwise, the baby will be red/sorrel with no Appaloosa spotting). Suppose a genetic researcher analyzed a sample of 200 foals in which both the sire and dam were red/sorrel Appaloosas, and found that 135 of the foals were red/sorrel Appaloosas. Does this sample provide evidence that the proportion of Appaloosa foals from all such matings differs from 0.75? Use a 90% confidence level.
http://www.animalgenetics.us/Equine/CCalculator3.asp

Sample info

$n = 200$

$\hat{p} = \frac{135}{200} = 0.675$

Assumptions met

Spotted	Not Spotted
135	65

Both over 10

Hypotheses

p = proportion of babies that are Appaloosa spotted

$H_0: p = 0.75$

$H_1: p \neq 0.75$

$\alpha = 0.10$

benchmark: $p_0 = 0.75$

$1 - p_0 = 0.25$

Std error

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.75(0.25)}{200}}$$

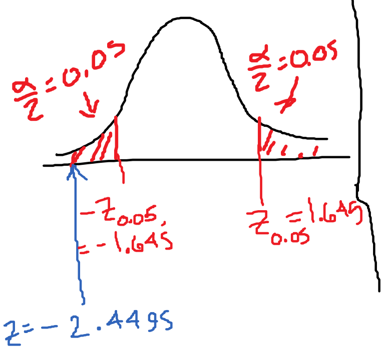
≈ 0.0306186 (stor in calculator)

Test statistic

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.675 - 0.75}{0.0306186}$$

≈ -2.4495

Picture



Find critical value:

Look in table w/ area = 0.95 $\Rightarrow z_{0.05} = 1.645$

Conclusion

$\boxed{\text{Reject } H_0}$

This sample provides evidence that the proportion of foals with Appaloosa spots differs from 0.75.