# 1342-Notes_Navidi_Set-Theory-supplement

Wednesday, October 2, 2019 9:25 AM

1342-Notes_Navidi_Set-Theory-supplement

## Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

The set of students officially enrolled in this class.
Set of students attended this class today
Set of all words in the 2019 Merriam-Webster English dictionary

Not sets:

All tall people (not well-defined)
All cute ~~dog~~ dogs

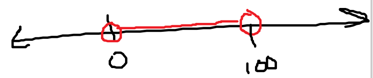
Sets can be finite or infinite.

Examples of finite sets:

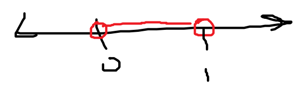
All the set examples above
All the integers between 0 and 100

Examples of infinite sets:

All the integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
All real numbers between 0 and 100.



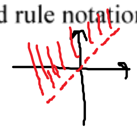

(or all real numbers in interval $(0,1)$)




Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.
- $a \in A$ means $a$ is an element of the set $A$. $a \in A$
  $a \notin A$ means $a$ is not an element of the set $A$. $a \notin A$
- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*. $\varnothing$
- $S = \{x \mid P(x)\}$ means "$S$ is the set of all $x$ such that $P(x)$ is true". (called rule notation or set roster notation).

  $S = \{(x,y) \mid x < y\}$

  Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \ldots\}$
- $n(A)$ means the number of elements in set $A$.

Definition: We say two sets are *equal* if they have exactly the same elements.

**Subsets:**

Definition: If each element of a set $A$ is also an element of set $B$, we say that $A$ is a *subset* of $B$. This is denoted $A \subseteq B$ or $A \subset B$. If $A$ is not a subset of $B$, we write $A \not\subset B$.

$A \subseteq B$ or $A \subset B$

Definition: We say $A$ is a *proper subset* of $B$ if $A \subseteq B$ but $A \neq B$. (In other words, every element of $A$ is also an element of $B$, but $B$ contains at least one element that is not in $A$.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted $U$.
Example: If you're dealing with sets of real numbers, then $U$ is the set of all real numbers. So "Wednesday" would not be an element of $U$, but 5.7 would be in $U$.

**Example 1:** Consider these sets.

$A = \{1,2,3,4,5,6\}$ $\quad A \subseteq B$

$B = \{1,2,3,4,5,6,7,8\}$ $\quad C \subseteq B$

$C = \{1,3,5,2,4,6\}$ $\quad A = C$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set $A$.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set $A$.)

**Example 2:** List all subsets of $\{1,2,3\}$.

$\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\varnothing$, $\{1,2,3\}$

Note: $n=3$
There are $2^3 = 8$ subsets

Note: If a set has $n$ elements, how many subsets does it have? $2^n$ subsets

**Set operations:**

- Union $\cup$ : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ (this is an "inclusive or".. could be in A or B or possibly both)
  Key Word: OR
- Intersection $\cap$ : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
  Key word: AND
- Complement $A'$ or $A^C$ or $\tilde{A}$: $A' = \{x \in U \mid x \notin A\}$.
  Key word NOT

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that $A$ and $B$ are *disjoint sets* if $A \cap B = \varnothing$.

(or mutually exclusive}

**Example 3:** $U = \{7,8,9,10,11,12,13,14,15\}$ = universal set

$H = \{7,9,11,13,15\}$

$K = \{10,11,12\}$

$J = \{8,10,12,14\}$

$L = \{9,14\}$

$m = \{10,11\}$

$J \cap K = \{10, 12\}$

$K \cup J = J \cup K = \{8, 10, 12, 14, 11\} = \{8, 10, 11, 12, 14\}$

$H \cup J = \{7, 8, 9, 10, 11, 12, 13, 14, 15\} = U$

$H \cap J = \varnothing$

$L \cup J = \{8, 9, 10, 12, 14\}$

$L \cap J = \{14\}$

$m \subseteq K$

$m \cup K = \{10, 11, 12\}$

$m \cap K = \{10, 11\}$

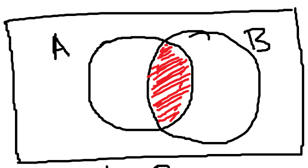
$K^c = K' = \{7, 8, 9, 13, 14, 15\}$

(K-complement)

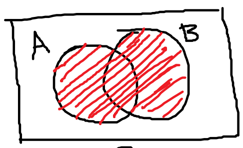
$(L \cup J)' = \{7, 11, 13, 15\}$

**Venn Diagrams:** These help us visualize set relationships and operations.

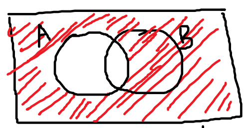
**Example 4:** Draw Venn diagrams for $A \cup B$, $A \cap B$, $A^C$, $B^C$, $(A \cap B)^C$, and $(A \cup B)^C$.



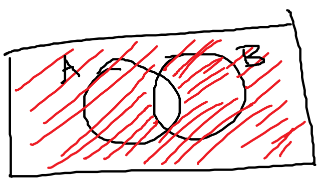

$A \cap B$




$A \cup B$

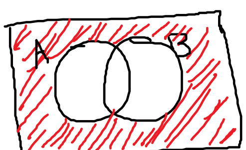



$A^c = A'$

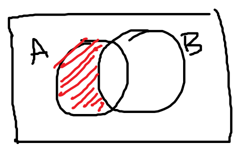



$(A \cap B)^c$

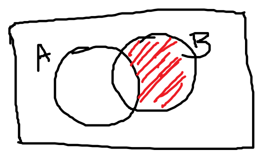
$= (A \cap B)'$




$(A \cup B)^c$




$A \cap B^c$




$B \cap A^c$

$= A^c \cap B$