Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

Not sets:

Sets can be finite or infinite.

Examples of finite sets:

Examples of infinite sets:

Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.
- $a \in A$ means a is an element of the set A.
  $a \notin A$ means a is not an element of the set A.
- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.
- $S = \{x \mid P(x)\}$ means "S is the set of all x such that $P(x)$ is true". (called rule notation or set roster notation).

  Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \ldots\}$
- $n(A)$ means the number of elements in set A.

Definition: We say two sets are *equal* if they have exactly the same elements.

Subsets:

Definition: If each element of a set *A* is also an element of set *B*, we say that *A* is a *subset* of *B*. This is denoted $A \subseteq B$ or $A \subset B$. If *A* is not a subset of *B*, we write $A \not\subset B$.

Definition: We say *A* is a *proper subset* of *B* if $A \subseteq B$ but $A \neq B$. (In other words, every element of *A* is also an element of *B*, but *B* contains at least one element that is not in *A*.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted *U*.

Example: If you're dealing with sets of real numbers, then *U* is the set of all real numbers. So "Wednesday" would not be an element of *U*, but 5.7 would be in *U*.

Example 1: Consider these sets.

$$A = \{1,2,3,4,5,6\}$$
$$B = \{1,2,3,4,5,6,7,8\}$$
$$C = \{1,3,5,2,4,6\}$$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set *A*.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set *A*.)

Example 2: List all subsets of $\{1,2,3\}$.

Note: If a set has *n* elements, how many subsets does it have?

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement A' or A^C or $A^{\square}$: $A' = \{x \in U \mid x \notin A\}$.

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.
$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that *A* and *B* are *disjoint sets* if $A \cap B = \varnothing$.

Example 3: $U = \{7,8,9,10,11,12,13,14,15\}$
$H = \{7,9,11,13,15\}$
$K = \{10,11,12\}$
$J = \{8,10,12,14\}$
$L = \{9,14\}$

Venn Diagrams: These help us visualize set relationships and operations.

Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.