1314-1-6-Notes-Other-Eqns

Wednesday, September 25, 2019 11:21 AM

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1.6: Other Types of Equations

Solving polynomial equations by factoring:

Example 1: Solve $9x^2 + 20x = -x^3$.

$+x^3 \qquad +x^3$

This is a cubic equation

$$x^3 + 9x^2 + 20x = 0$$
$$x(x^2 + 9x + 20) = 0$$
$$x(x+4)(x+5) = 0$$
$$x = 0 \mid x+4=0 \mid x+5=0$$
$$x = -4 \qquad x = -5$$

Sol'n Set: $\boxed{\{0, -4, -5\}}$

Example 2: Solve $x^3 + 5x^2 - 9x = 45$.

Note:
Ex: $x^2y - 9y = 0$
$y(x^2 - 9) = 0$

$$x^3 + 5x^2 - 9x - 45 = 0$$
$$(x^3 + 5x^2) + (-9x - 45) = 0$$
$$x^2(x+5) - 9(x+5) = 0$$
$$(x+5)(x^2 - 9) = 0$$

$$(x+5)(x+3)(x-3) = 0$$
$$x+5=0 \mid x+3=0 \mid x-3=0$$
$$x=-5 \mid x=-3 \mid x=3$$

Sol'n Set: $\boxed{\{-5, \pm 3\}}$

Example 3: Solve $x^3 = 16x$.

Example 4: Solve $2x^4 = 32x^2$.

$$2x^4 - 32x^2 = 0$$
$$2x^2(x^2 - 16) = 0$$
$$2x^2(x+4)(x-4) = 0$$
$$2x \cdot x(x+4)(x-4) = 0$$
$$2x = 0 \mid x = 0 \mid x+4=0 \mid x-4=0$$
$$\frac{2x}{2} = \frac{0}{2} \mid x = -4 \mid x = 4$$
$$x = 0$$

Sol'n Set: $\boxed{\{0, \pm 4\}}$

or $\boxed{\{0, -4, 4\}}$

Example 5: Solve $x^5 = x$

Solving equations involving radicals:

A radical equation is one in which the variable appears in a root, or radical sign. It may be a square root, cube root, or higher root. To solve these, we must isolate the radical and then raise both sides to a power.

Example 6: Solve $\sqrt{x} = 4$.

$(\sqrt{x})^2 = (4)^2$

$x = 16$

Sol'n Set: $\boxed{\{16\}}$

Check: $\sqrt{x} = 4$

$x = 16 \Rightarrow \sqrt{16} = 4$

$4 = 4$ ✓

Example 7: Solve $\sqrt[5]{x-7} = -2$

$\left(\sqrt[5]{x-7}\right)^5 = (-2)^5$

$\left((x-7)^{1/5}\right)^5 = (-2)^5$

$x - 7 = -32$

$+7 \quad +7$

$x = -25$

Sol'n Set: $\boxed{\{-25\}}$

Example 8: Solve $\sqrt{2-x}-10=x$.

$+10 \qquad +10$

[Isolate the radical]

$\sqrt{2-x} = x+10$

$\left(\sqrt{2-x}\right)^2 = (x+10)^2$

[Square both sides]

$2-x = (x+10)(x+10)$

$2-x = x^2+10x+10x+100$

$2-x = x^2+20x+100$

$-2 \ +x \qquad +x \qquad -2$

$0 = x^2+21x+98$

$0 = (x+7)(x+14)$

$x+7=0 \quad | \quad x+14=0$

$x=-7 \quad | \quad x=-14$

Check your answers!

Let's check our answers

$\sqrt{2-x}-10=x$

$x=-7$: $\sqrt{2-(-7)}-10=-7$

$\sqrt{2+7}-10=-7$

$\sqrt{9}-10=-7$

$3-10=-7$

$-7=-7$ ✓ ok

$x=-14$: $\sqrt{2-(-14)}-10=-14$

$\sqrt{2+14}-10=-14$

$\sqrt{16}-10=-14$

$4-10=-14$

$-6=-14$

False!!

Throw out $x=-14$

$\boxed{\text{Sol'n Set is } \{-7\}}$

<u>Extraneous Solutions</u>:
Raising both sides of an equation to an even power can introduce *extraneous solutions*, or *extraneous roots*, that are not solutions to the original equation.

Important: Whenever you square both sides, or raise both sides to any even power, you must check your solutions in the original equation.

<u>Extraneous Solution</u>: A value generated by a correct solution process that doesn't check in the original equation.

Example 9: Solve $x-\sqrt{3-x}=-3$.

$+\sqrt{3-x} \qquad +\sqrt{3-x}$

$x = -3+\sqrt{3-x}$

$+3 \qquad +3$

$x+3 = \sqrt{3-x}$

$(x+3)^2 = \left(\sqrt{3-x}\right)^2$

$(x+3)(x+3) = 3-x$

$x^2+6x+9 = 3-x$

$+x \quad -3 \qquad -3 \quad +x$

$x^2+7x+6=0$

$(x+6)(x+1)=0$

$x+6=0 \quad | \quad x+1=0$

$x=-6 \quad | \quad x=-1$

OR

$x-\sqrt{3-x}=-3$

$-x \qquad -x$

$-\sqrt{3-x}=-3-x$

$\sqrt{3-x}=3+x$ (Divide by -1)

Check answers (mandatory!)

$x-\sqrt{3-x}=-3$

$x=-6 \Rightarrow -6-\sqrt{3-(-6)}=-3$

$-6-\sqrt{9}=-3$

$-6-3=-3$

$-9=-3$ False!

Throw out $x=-6$

$x-\sqrt{3-x}=-3$

$x=-1 \Rightarrow -1-\sqrt{3-(-1)}=-3$

$-1-\sqrt{3+1}=-3$

$-1-\sqrt{4}=-3$

$-1-2=-3$

$-3=-3$ true!

Sol's Set: $\boxed{\{-1\}}$

$9x^2$ $20x$ 80

$9x \cdot x$ | $1 \cdot 80$
$3x \cdot 3x$ | $2 \cdot 40$
$36x$ | $4 \cdot 20$
$5 \cdot 16$
$8 \cdot 10$

Example 10: Solve $\sqrt{1-2x} = 4 - \sqrt{x+5}$.

(Isolate the more complicated one)

1st, isolate one of the radicals.

$$\sqrt{1-2x} = 4-\sqrt{x+5}$$
$$\left(\sqrt{1-2x}\right)^2 = \left(4-\sqrt{x+5}\right)^2$$
$$1-2x = \left(4-\sqrt{x+5}\right)\left(4-\sqrt{x+5}\right)$$
$$1-2x = 16 - 4\sqrt{x+5} - 4\sqrt{x+5} + x+5$$
$$1-2x = 16 - 8\sqrt{x+5} + x+5$$
$$-16 \quad -x \qquad -16 \qquad -x \; -5$$

Isolate the remaining radical: $-20-3x = -8\sqrt{x+5}$

$$(-20-3x)^2 = \left(-8\sqrt{x+5}\right)^2$$
$$(-20-3x)(-20-3x) = 64(x+5)$$
$$400+60x+60x+9x^2 = 64x+320$$
$$400+120x+9x^2 = 64x+320$$
$$-320 \quad -64x \qquad -64x \quad -320$$
$$9x^2+56x+80=0$$
$$(9x+20)(x+4)=0$$
$$9x+20=0 \quad | \quad x+4=0$$
$$9x=-20 \qquad x=-4$$
$$x = -\frac{20}{9}$$

Check your answers!
Both answers check,
so Solution Set is
$$\boxed{\left\{-\frac{20}{9}, -4\right\}}$$

Solving equations with rational exponents:

Example 11: Solve $x^{2/3}+4=9$.

$$x^{\frac{2}{3}}+4=9$$
$$-4 \quad -4$$
$$x^{\frac{2}{3}} = 5$$
$$\left(x^2\right)^{1/3} = 5$$
$$\sqrt[3]{x^2} = 5$$
$$\left(\sqrt[3]{x^2}\right)^3 = (5)^3$$
$$x^2 = 125$$
$$\sqrt{x^2} = \pm\sqrt{125}$$
$$x = \pm\sqrt{125}$$
$$x = \pm\sqrt{25\cdot 5}$$
$$x = \pm 5\sqrt{5}$$

Solution Set: $\boxed{\left\{\pm 5\sqrt{5}\right\}}$

Note: $x^{\frac{2}{3}} = \left(x^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{x}\right)^2$

also $x^{\frac{2}{3}} = \left(x^2\right)^{\frac{1}{3}} = \sqrt[3]{x^2}$

Recall: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$\left(x^a\right)^b = x^{ab}$

Example 12: Solve $2x^{3/2}=54$.

$$2x^{\frac{3}{2}} = 54$$
$$\frac{2x^{\frac{3}{2}}}{2} = \frac{54}{2}$$
$$x^{\frac{3}{2}} = 27$$
$$\left(x^{\frac{1}{2}}\right)^3 = 27$$
$$\left(\sqrt{x}\right)^3 = 27$$

Take cube root of both sides:
$$\sqrt{x} = \sqrt[3]{27}$$
$$\sqrt{x} = 3$$

Square both sides:
$$\left(\sqrt{x}\right)^2 = (3)^2$$
$$x = 9$$

Soln Set: $\boxed{\{9\}}$

Check your answers:
$$2x^{\frac{3}{2}} = 54$$
$$2(9)^{3/2} = 54$$
$$2\left(9^{\frac{1}{2}}\right)^3 = 54$$
$$2\left(\sqrt{9}\right)^3 = 54$$
$$2(3)^3 = 54$$
$$2(27) = 54$$
$$54 = 54 \text{ True!}$$

Solving equations quadratic in form:

An equation is called *quadratic in form* if, through an appropriate substitution, it can be rewritten as a quadratic equation in another variable. (instead of being a quadratic in x, it is a quadratic in something else)

Example 13: Solve $x^4 - 7x^2 - 8 = 0$.

$x^4 - 7x^2 - 8 = 0$
$(x^2)^2 - 7x^2 - 8 = 0$
$u = x^2 \Rightarrow (u)^2 - 7u - 8 = 0$
$u^2 - 7u - 8 = 0$
$(u - 8)(u + 1) = 0$
$u - 8 = 0 \mid u + 1 = 0$
$u = 8 \mid u = -1$
$u = x^2 \Rightarrow x^2 = 8 \mid x^2 = -1$

$x^2 = 8 \mid x^2 = -1$
$x = \pm\sqrt{8} \mid x = \pm\sqrt{-1}$
$x = \pm\sqrt{4 \cdot 2}$
$x = \pm 2\sqrt{2} \mid x = \pm i$

Sol'n Set: $\boxed{\{\pm 2\sqrt{2}, \pm i\}}$

Note: You don't have to write the u's:
$x^4 - 7x^2 - 8 = 0$
$(x^2 - 8)(x^2 + 1) = 0$
$x^2 - 8 = 0 \mid x^2 + 1 = 0$
$x^2 = 8 \mid x^2 = -1$
$x = \pm\sqrt{8} \mid x = \pm\sqrt{-1}$
$= \pm i$

Example 14: Solve $2x^6 - x^3 = 3$.
$ -3 \quad -3$

$2x^6 - x^3 - 3 = 0$
$2(x^3)^2 - (x^3) - 3 = 0$
$u = x^3 \Rightarrow 2u^2 - u - 3 = 0$
$(2u - 3)(u + 1) = 0$
$2u - 3 = 0 \mid u + 1 = 0$
$2u = 3 \mid u = -1$
$\frac{2u}{2} = \frac{3}{2}$
$u = \frac{3}{2}$

$u = \frac{3}{2} \mid u = -1$
$u = x^3 \Rightarrow x^3 = \frac{3}{2} \mid x^3 = -1$
$\sqrt[3]{x^3} = \sqrt[3]{\frac{3}{2}} \mid \sqrt[3]{x^3} = \sqrt[3]{-1}$
$x = \sqrt[3]{\frac{3}{2}} \mid x = -1$

$x = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$

$x = \frac{\sqrt[3]{3}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{12}}{\sqrt[3]{8}} = \frac{\sqrt[3]{12}}{2}$

Note: $\sqrt[3]{-1} = -1$ because $(-1)^3 = -1$

Sol'n Set: $\boxed{\left\{\frac{\sqrt[3]{12}}{2}, -1\right\}}$

Example 15: Solve $x^{-2} + 5x^{-1} + 6 = 0$.

$(x^{-1})^2 + 5x^{-1} + 6 = 0$
$(\boxed{u})^2 + 5\boxed{u} + 6 = 0$
$u = x^{-1} \Rightarrow u^2 + 5u + 6 = 0$
$(u + 2)(u + 3) = 0$
$u + 2 = 0 \mid u + 3 = 0$
$u = -2 \mid u = -3$
$u = x^{-1} = \frac{1}{x} \Rightarrow \frac{1}{x} = -2 \mid \frac{1}{x} = -3$

$\frac{1}{x} = -2 \mid \frac{1}{x} = -3$
$\frac{1}{x} = \frac{-2}{1} \mid \frac{1}{x} = \frac{-3}{1}$
$1 = -2x \mid 1 = -3x$
$\frac{1}{-2} = \frac{-2x}{-2} \mid \frac{1}{-3} = \frac{-3x}{-3}$
$-\frac{1}{2} = x \mid -\frac{1}{3} = x$

Sol'n Set: $\boxed{\left\{-\frac{1}{2}, -\frac{1}{3}\right\}}$

$81 \cdot 9 = 729$

Example 16: Solve $x^{2/3} - 5x^{1/3} = 36$.

$x^{\frac{2}{3}} - 5x^{\frac{1}{3}} = 36$

$\left(x^{\frac{1}{3}}\right)^2 - 5\left(x^{\frac{1}{3}}\right) - 36 = 0$

$u = x^{\frac{1}{3}} \Rightarrow u^2 - 5u - 36 = 0$

$(u-9)(u+4) = 0$

$u - 9 = 0 \mid u + 4 = 0$

$u = 9 \mid u = -4$

$u = x^{\frac{1}{3}} \Rightarrow$

$u = 9$	$u = -4$
$x^{\frac{1}{3}} = 9$	$x^{\frac{1}{3}} = -4$
$\sqrt[3]{x} = 9$	$\sqrt[3]{x} = -4$
$\left(\sqrt[3]{x}\right)^3 = (9)^3$	$\left(\sqrt[3]{x}\right)^3 = (-4)^3$
$x = 729$	$x = -64$

Sol'n Set: $\boxed{\{729, -64\}}$

Solving absolute value equations:

Absolute value is another way of saying "size" or "magnitude" or "distance from zero".

For example, $|-5|$ is the distance from -5 to 0 on the number line.
$|x|$ is the distance from x to 0 on the number line.
So if we say that $|x| = 8$, then it must be that $x = 8$ or $x = -8$.

5 5
-5 -4 -3 -2 -1 0 1 2 3 4 5

Recall:
$|6| = 6$
$|-6| = 6$
$|0| = 0$

Ex: Solve $|x| = 8$

$x = 8, x = -8$

-8 0 8

Sol'n Set: $\boxed{\{\pm 8\}}$

To solve absolute value equations:

- Isolate the absolute value on one side.
- If C is positive, use the fact that $|x| = C$ means that $x = \pm C$ to write two new equations.
- Solve each of these resulting equations.

Example 17: Solve $|2x - 3| - 7 = 0$.

+7 +7

$|2x - 3| = 7$

[Isolate the absolute value]

2x-3 2x-3

-7 0 7

$2x - 3 = -7$ OR $2x - 3 = 7$

+3 +3 | +3 +3

$2x = -4$	$2x = 10$
$\frac{2x}{2} = \frac{-4}{2}$	$\frac{2x}{2} = \frac{10}{2}$
$x = -2$	$x = 5$

Sol'n Set: $\boxed{\{-2, 5\}}$

Check: $|2x - 3| - 7 = 0$

$x = -2 \Rightarrow |2(-2) - 3| - 7 = 0$

$|-4 - 3| - 7 = 0$

$|-7| - 7 = 0$

$7 - 7 = 0$

$0 = 0$

Example 18: Solve $2|5-2x|+6=14$.

Example 19: Solve $20+|2x+4|=15$.

Example 20: Solve $|x+3|=|2x+1|$.

Example 21: Solve $\frac{1}{|x+3|}=2$.