# 1314-2-5-Notes-transformations

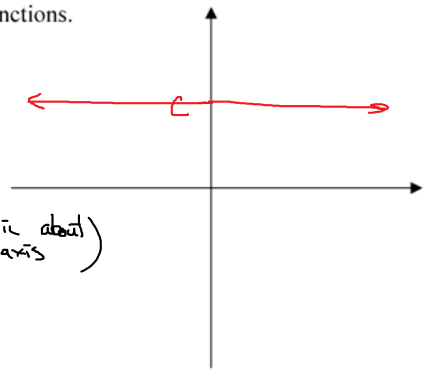
Thursday, September 26, 2019 10:52 AM

1314-2-5-Notes-transformations

## 2.5: Transformations of Functions

Learn the graphs of these frequently encountered functions.

Constant function $f(x) = c$




Domain: $(-\infty, \infty)$

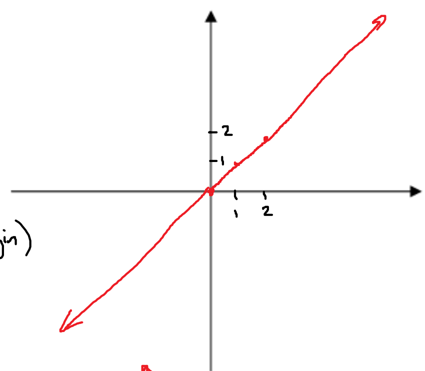
Range: $\{c\}$

Is it odd or even? even function (symmetric about y-axis)

Ex. $f(x) = -2$

| $x$ | $f(x)$ |
|---|---|
| $-3$ | $f(-3) = -2 \Rightarrow (-3,-2)$ |
| $-2$ | $f(-2) = -2 \Rightarrow (-2,-2)$ |
| $-1$ | $f(-1) = -2$ |
| $0$ | $f(0) = -2$ |
| $1$ | $f(1) = -2$ |
| $2$ | $-2$ |
| $3$ | $-2$ |

Identity function $f(x) = x$




Domain: $(-\infty, \infty)$

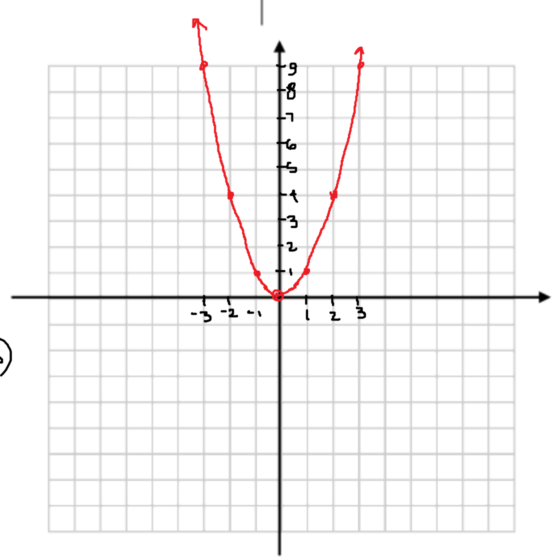
Range: $(-\infty, \infty)$

Is it odd or even? odd function (symmetric about origin)

ordered pairs:
$(0,0), (1,1), (2,2), (3,3)$
$(-2,-2), (-5,-5)$

OR, using $y = mx + b$
$y = x$
$y = 1x + 0 \Rightarrow$ slope: $m = 1 = +\frac{1}{1}$
y-int: $b = 0 \Rightarrow (0,0)$

Standard quadratic function $f(x) = x^2$




Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even? even function (symmetric around y-axis)

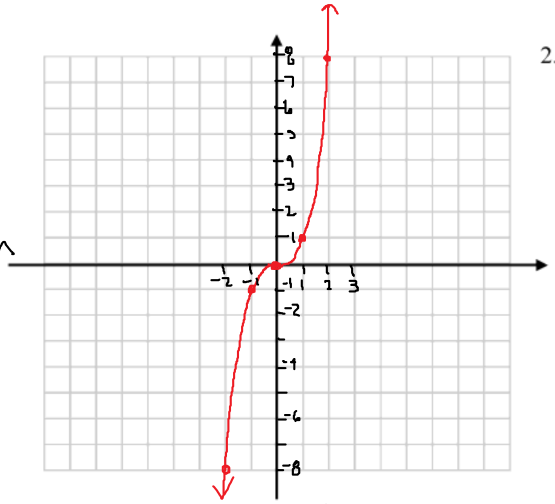
| $x$ | $f(x) = x^2$ |
|---|---|
| $-3$ | $(-3)^2 = 9$ |
| $-2$ | $(-2)^2 = 4$ |
| $-1$ | $(-1)^2 = 1$ |
| $0$ | $(0)^2 = 0$ |
| $1$ | $(1)^2 = 1$ |
| $2$ | $(2)^2 = 4$ |
| $3$ | $(3)^2 = 9$ |

Standard cubic function $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Is it odd or even? odd function (symmetric about the origin)



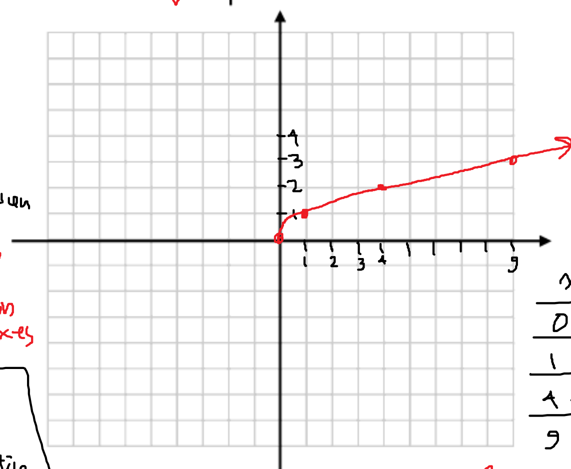
| $x$ | $f(x) = x^3$ |
|---|---|
| $-3$ | $(-3)^3 = -27$ |
| $-2$ | $(-2)^3 = -8$ |
| $-1$ | $(-1)^3 = -1$ |
| $0$ | $(0)^3 = 0$ |
| $1$ | $(1)^3 = 1$ |
| $2$ | $(2)^3 = 8$ |
| $3$ | $(3)^3 = 27$ |

Square root function $f(x) = \sqrt{x}$ :

Domain: $[0, \infty)$

Range: $[0, \infty)$

Is it odd or even? neither odd nor even



| $x$ | $f(x) = \sqrt{x}$ |
|---|---|
| $-3$ | $\sqrt{-3} = i\sqrt{3}$ not a real number |
| $-2$ | not real |
| $-1$ | not real |
| $0$ | $f(0) = \sqrt{0} = 0$ |
| $1$ | $f(1) = \sqrt{1} = 1$ |
| $4$ | $f(4) = \sqrt{4} = 2$ |
| $9$ | $f(9) = \sqrt{9} = 3$ |

not valid inputs

cannot be used for graphing on x and y axes

In sections on graphing functions, we will only consider real-valued functions

know these

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

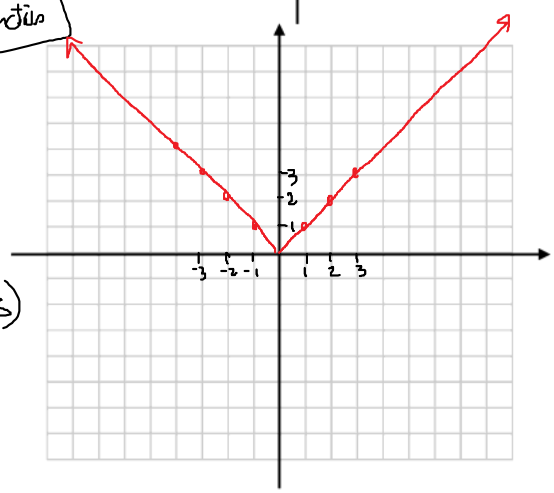
Absolute value function $f(x) = |x|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Is it odd or even?

even function (symmetric around y-axis)



| $x$ | $f(x) = \lvert x \rvert$ |
|---|---|
| $-3$ | $\lvert -3 \rvert = 3$ |
| $-2$ | $\lvert -2 \rvert = 2$ |
| $-1$ | $\lvert -1 \rvert = 1$ |
| $0$ | $\lvert 0 \rvert = 0$ |
| $1$ | $\lvert 1 \rvert = 1$ |
| $2$ | $\lvert 2 \rvert = 2$ |
| $3$ | $\lvert 3 \rvert = 3$ |

If the graph of a function is known, similar functions can be graphed by varying them in several ways.

- Vertical translation (shifting vertically)
- Horizontal translation (shifting horizontally)
- Reflecting about the $x$-axis
- Reflecting about the $y$-axis
- Vertical stretching and shrinking

**Translation of functions:**

To *translate* a graph means to shift it horizontally, vertically, or both.

Horizontal Translation:

- Replacing $x$ by $x-c$, with $c$ positive, shifts the graph $c$ units to the right.
- Replacing $x$ by $x+c$, with $c$ positive, shifts the graph $c$ units to the left.

Note: $x+c=0$ | $x-c=0$
$x=-c$ | $x=c$

Vertical Translation:

- Adding a positive number $d$ to the function shifts the graph upward by $d$ units.
- Subtracting a positive number $d$ from the function shifts the graph downward by $d$ units.

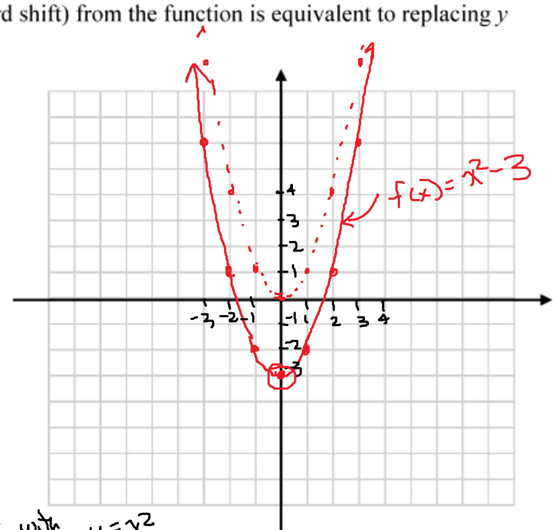
Note:

- Adding a positive number $d$ to the function (upward shift) is equivalent to replacing $y$ by $y-d$.

$$y = f(x)+d$$
$$y-d = f(x)$$

- Subtracting a positive number $d$ (downward shift) from the function is equivalent to replacing $y$ by $y+d$.

$$y = f(x)-d$$
$$y+d = f(x)$$

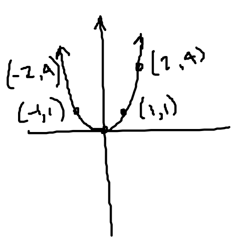



**Example 1:** Sketch the graph of $f(x) = x^2 - 3$.

Determine the "parent function" this is created from

$y = x^2$

| $x$ | $y=x^2$ |
|---|---|
| 3 | 9 |
| 2 | 4 |
| 1 | 1 |
| 0 | 0 |
| -1 | 1 |
| -2 | 4 |
| -3 | 9 |

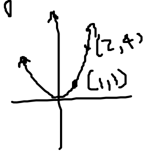



Start with $y=x^2$
Shift (translate) it Down 3

**Example 2:** Sketch the graph of $g(x)=(x+2)^2$.

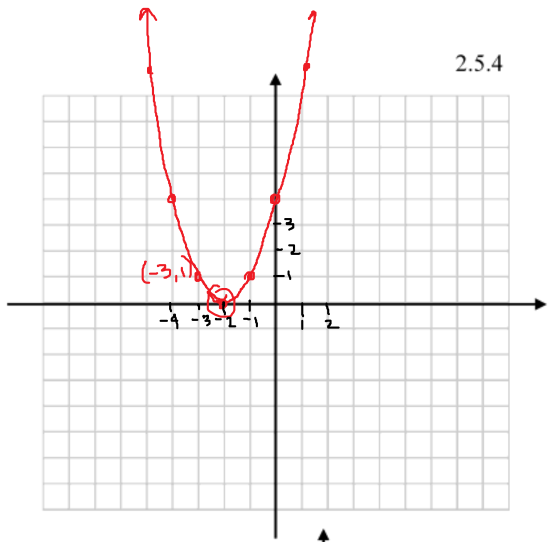
Parent function:

$y=x^2$ , $(3,9)$



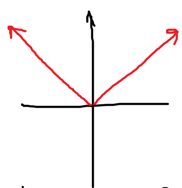

Start with $y=x^2$.
Then shift it left 2.

Why move left?
$x+2=0$
$x=-2$

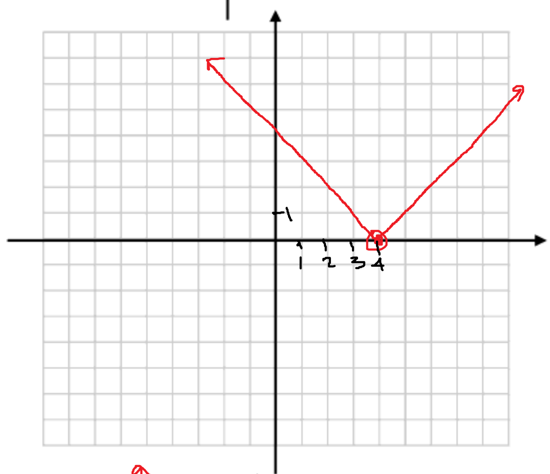



**Example 3:** Sketch the graph of $f(x)=|x-4|$.

Parent function: $y=|x|$

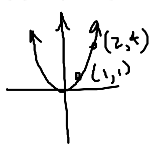


Start with $y=|x|$ then shift it right 4
(because $x-4=0$
$x=4$)



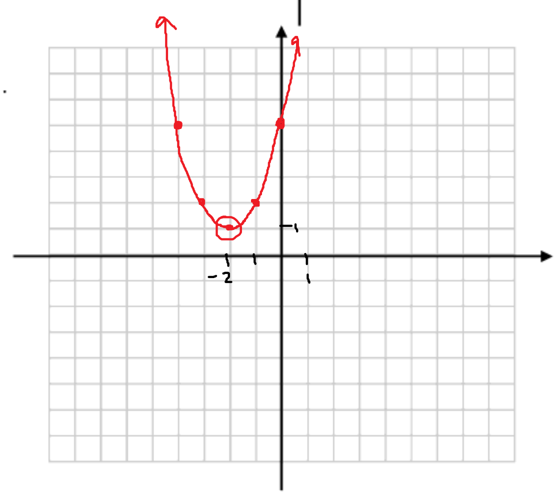
**Example 4:** Sketch the graph of $f(x)=(x+2)^2+1$.

Parent function: $y=x^2$
shift it left 2, up 1



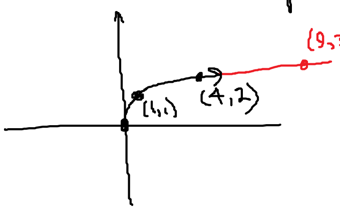

$y=(x+2)^2+1$
$y-1=(x+2)^2$

$x+2=0$
$x=-2$
$y-1=0$
$y=1$



**Example 5:** Sketch the graph of $y=\sqrt{x-2}-3$.

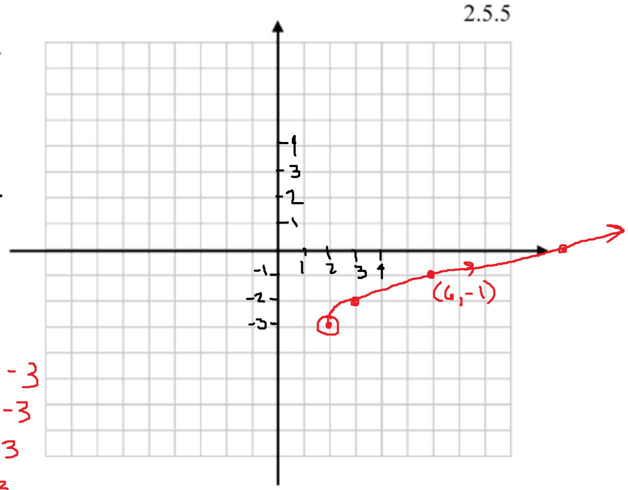
Parent function: $y=\sqrt{x}$




shift it: right 2

Note: $x-2=0$
$x=2$

down 3




Check. Is $(6,-1)$ on the graph of $y=\sqrt{x-2}-3$

$x=6 \Rightarrow y=\sqrt{6-2}-3$
$y=\sqrt{4}-3$
$y=2-3$
$y=-1 \Rightarrow$ yes $(6,-1)$ on graph

**Reflection of functions:**

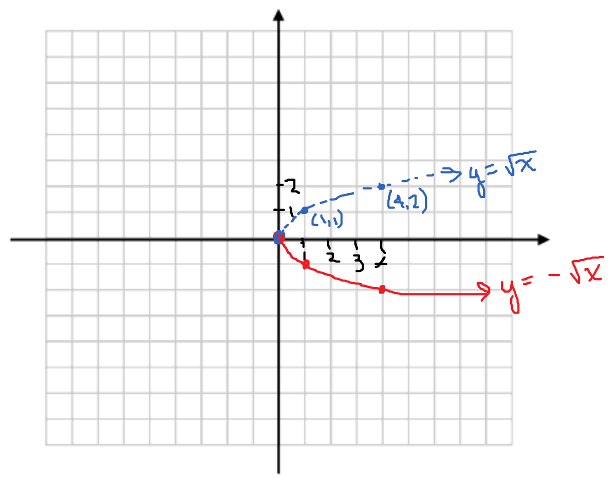
A *reflection* is the "mirror-image" of graph about the $x$-axis or $y$-axis.

- Changing $f(x)$ to $-f(x)$ (multiplying $f(x)$ by $-1$ reflects the graph about the $x$-axis. (e.g. changing $y=\sqrt{x}$ to $y=-\sqrt{x}$ )
- Changing $f(x)$ to $f(-x)$ (replacing $x$ by $-x$ ) reflects the graph about the $y$-axis. (e.g. changing $y=\sqrt{x}$ to $y=\sqrt{-x}$ )

**Example 6:** Sketch the graph of $f(x)=-\sqrt{x}$.

Parent Function: $y=\sqrt{x}$

Reflect it across $x$-axis



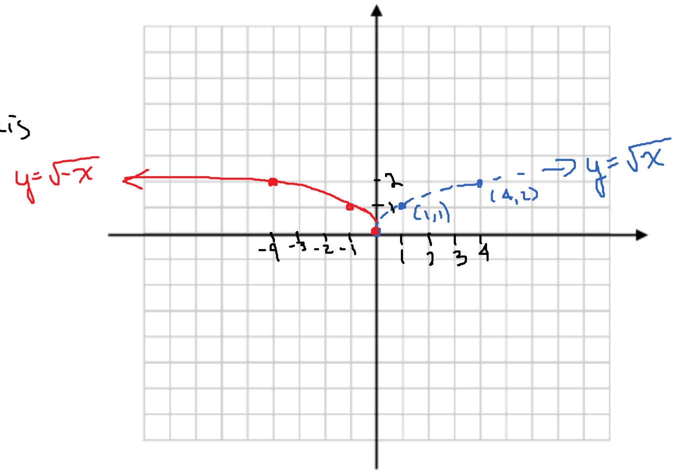
**Example 7:** Sketch the graph of $f(x)=\sqrt{-x}$.

Parent function: $y=\sqrt{x}$

Reflect it around the y-axis

Domain of $f(x)=\sqrt{-x}$ : $(-\infty, 0]$

Range of $f(x)=\sqrt{-x}$ : $[0,\infty)$




**Vertical stretching and shrinking:**

Multiplying a function by a number $a$ will multiply all the $y$-values by $a$.

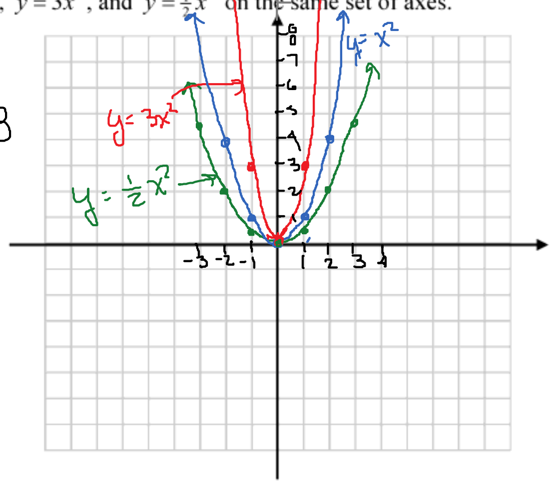
- Multiplying a function by $a$, with $a>1$, "stretches" the graph vertically.
- Multiplying a function by $a$, with $0<a<1$, "shrinks" the graph vertically.

**Example 8:** Sketch the graphs of $y=x^2$, $y=3x^2$, and $y=\frac{1}{2}x^2$ on the same set of axes.

All have parent function $y=x^2$

For $y=3x^2$: multiply the y-values by 3

For $y=\frac{1}{2}x^2$, start with $y=x^2$ and multiply the y-values by $\frac{1}{2}$



**Combinations of transformations:**

Recommended order for transformations:

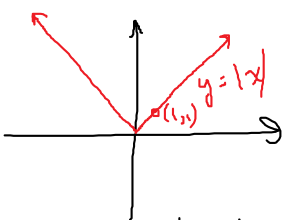
1. Stretching and shrinking
2. Reflection about $x$-axis.
3. Translations (horizontal and vertical).
4. Reflection about $y$-axis. (important to do this last!)

This is not the only order that works, but it is safest, and avoids having to perform algebraic manipulation of negative signs, etc.

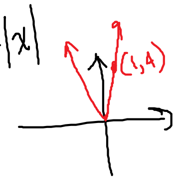
Note: $|x-3| \neq x+3$

**Example 9:** Sketch the graph of $f(x) = 4|x-3|+1$.

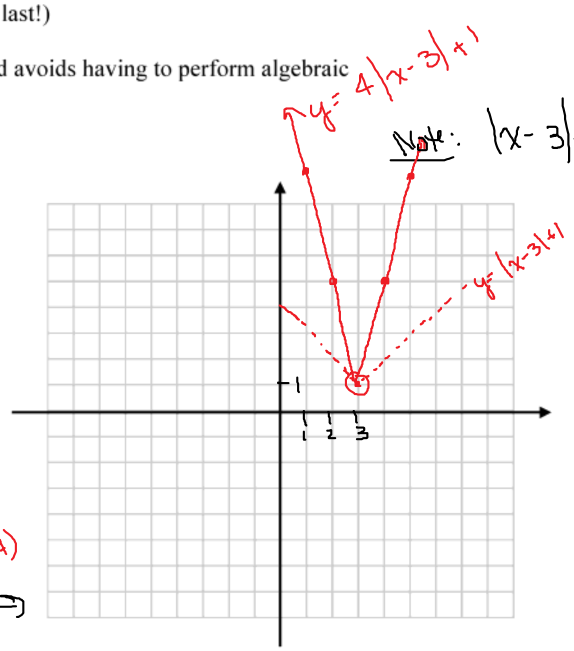
Parent function: $y = |x|$




Multiply y-values by 4: $y = 4|x|$



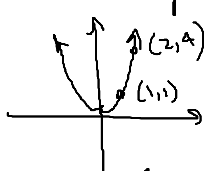

Shift this right 3, up 1



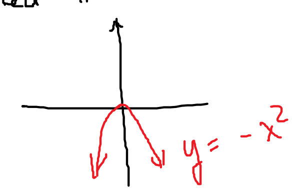

**Example 10:** Sketch the graph of $f(x) = -(x+2)^2 + 4$.

Parent function: $y = x^2$

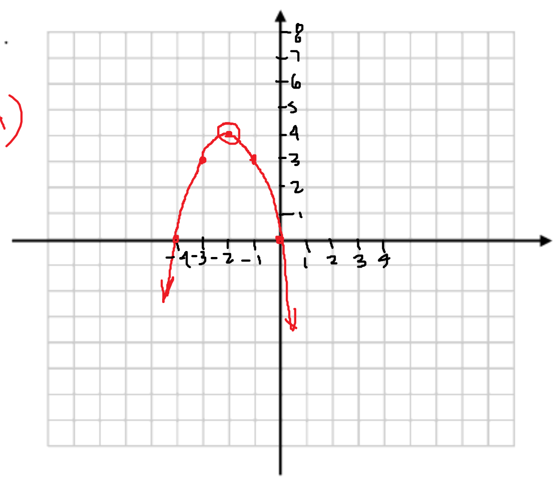
Vertex: $(-2,4)$




Then reflect it around x-axis:



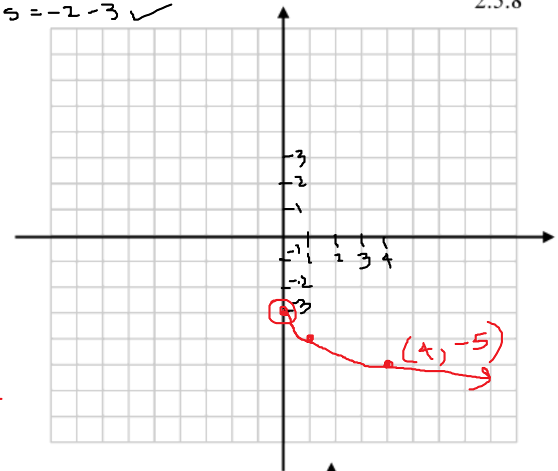

Then shift it left 2, up 4



Spot-check: Is $(4,-5)$ on the graph?
Graph: Set of ordered pairs that make the equation true.

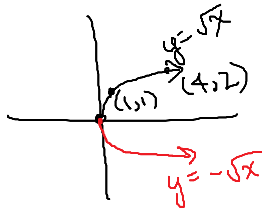
$y=-\sqrt{x}-3$
$-5=-\sqrt{4}-3$
$-5=-2-3$ ✓

**Example 11:** Sketch the graph of $y=-\sqrt{x}-3$.

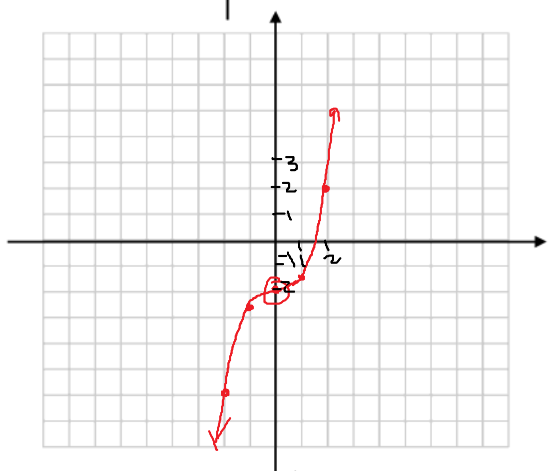



~~Basic~~ parent function: $y=\sqrt{x}$

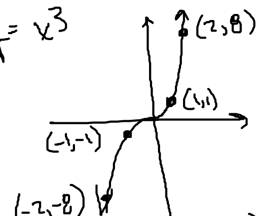
Reflect around x-axis
shift it down 3




**Example 12:** Sketch the graph of $y=\frac{1}{2}x^3-2$.



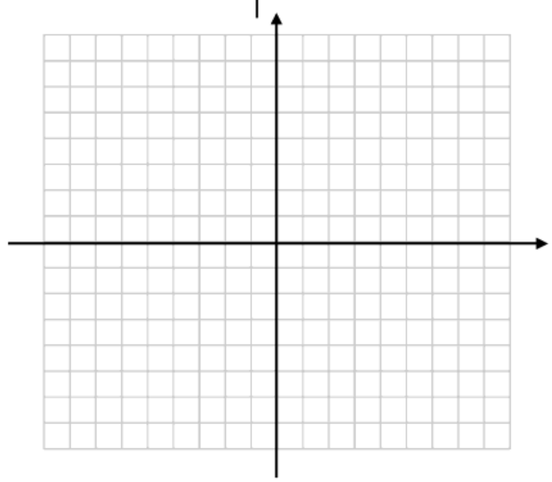
Parent function: $y=x^3$




Then multiply our y-values by $\frac{1}{2} \Rightarrow (1,\frac{1}{2})$
$(2,4)$

Then shift down 2

**Example 13:** Sketch the graph of $g(x)=-(x+1)^3-2$.

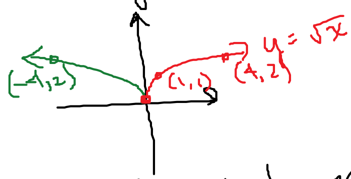


**Example 14:** Sketch the graph of $y = \sqrt{5-x} + 2$.

$y = \sqrt{-x+5} + 2$

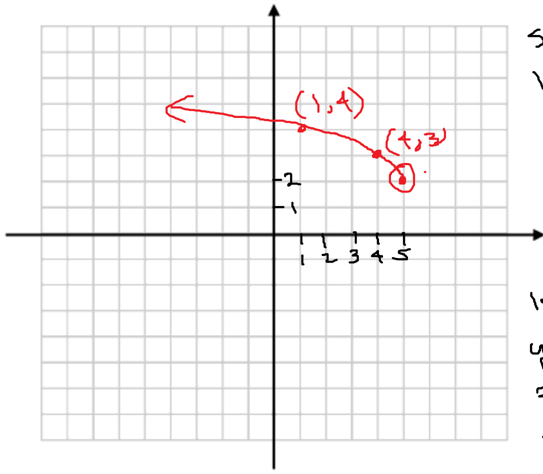
$y = \sqrt{-(x-5)} + 2$

Parent function: $y = \sqrt{x}$




If we then replace $x$ by $x-5$, that shifts it right 5.

The $+2$ at the end shifts it up 2.




Spot-check:

Is (1, 4) on graph?

$y = \sqrt{5-x} + 2$

$4 = \sqrt{5-1} + 2$

$4 = \sqrt{4} + 2$

$4 = 2+2$ ✓

Is (4, 3) on graph?

$y = \sqrt{5-x} + 2$

$3 = \sqrt{5-4} + 2$

$3 = \sqrt{1} + 2$

$3 = 1+2$

$3 = 3$ ✓

**Example 15:** Given the graph of $f(x)$, sketch the graph of

a. $-f(x)$

Reflect around $x$-axis (we're replacing $y$ by $-y$)

b. $f(-x)$

Reflect $f(x)$ around $y$-axis

c $f(x-2)$

Shift $f(x)$ right 2